

0344

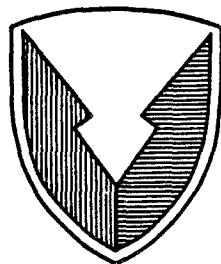
1027 ~~1027~~

A198067

**R** CENTER  
and LABORATORY  
TECHNICAL REPORT

NO. 13121

**IMPROVED FOUNDRY CASTINGS  
UTILIZING CAD/CAM  
VOLUME I: OVERVIEW**



**DAAK-30-78C-0107**

**JUNE 1985**

by CASTING INSTITUTE  
UNIVERSITY OF PITTSBURGH

APPROVED FOR PUBLIC RELEASE:  
DISTRIBUTION IS UNLIMITED.

**U.S. ARMY TANK-AUTOMOTIVE COMMAND  
RESEARCH AND DEVELOPMENT CENTER  
Warren, Michigan 48397-5000**

20020723207

Reproduced From  
Best Available Copy

AN-347030

**NOTICES**

**This report is not to be construed as an official Department of the Army position.**

**Mention of any trade names or manufacturers in this report shall not be construed as an official endorsement or approval of such products or companies by the U.S. Government.**

**Destroy this report when it is no longer needed. Do not return it to the originator.**

REPORT DOCUMENTATION PAGE				
1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED		1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release Distribution unlimited		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE				
4. PERFORMING ORGANIZATION REPORT NUMBER(S) SETEC-MME-84-019		5. MONITORING ORGANIZATION REPORT NUMBER(S) 13121, Volume 1		
6a. NAME OF PERFORMING ORGANIZATION University of Pittsburgh Casting Institute	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION U.S. Army TACOM R D + E Center		
6c. ADDRESS (City, State, and ZIP Code) 848 Benedum Hall Materials Science and Engineering Pittsburgh, PA 15261	7b. ADDRESS (City, State, and ZIP Code) AMSTA- TMM Warren, MI 48397-5000			
8a. NAME OF FUNDING/SPONSORING ORGANIZATION U.S. Army TACOM	8b. OFFICE SYMBOL (If applicable) AMSTA- TMM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER Contract DAAK-30-78-C-0107		
8c. ADDRESS (City, State, and ZIP Code) Warren, MI 48397-5000	10. SOURCE OF FUNDING NUMBERS			
	PROGRAM ELEMENT NO. 78011	PROJECT NO. DE 62	TASK NO. 4825014	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) Improved Foundry Castings Utilizing CAD/CAM: Volume I, Overview.				
12. PERSONAL AUTHOR(S) H.D. Brody, R.A. Stoehr, V. Srinath, W.S. Hwang, P. Wisniewski, N. Akgerman, A. Badawy, K. Fahey, S. Miller, R. Nariman, W. Northev, W. Schwarz				
13a. TYPE OF REPORT Final Technical	13b. TIME COVERED FROM 1978 Oct TO 1985 Jun	14. DATE OF REPORT (Year, Month, Day) 1985, June 30	15. PAGE COUNT 199	
16. SUPPLEMENTARY NOTATION 1988				
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) Computer assisted Manufacturing (CAD/CAM), Castings, Steel, Molds, Heat Flow, Fluid Flow, Finite Element, Finite Difference, Casting Soundness		
19. ABSTRACT (Continue on reverse if necessary and identify by block number) A software package, UPCAST, has been developed as an aid to casting engineers in designing and analyzing the manufacturing process for high integrity steel castings. The UPCAST routines use (1) computer graphics and computer assisted computations relative to heading and gating and (2) evaluation routines that allow 2-D and 3-D heat transfer analyses with a minimum of interaction from the user. Extensive experimental work was performed to validate the computer programs, to collect thermal properties needed for modeling steel castings in sand molds, and to predict casting soundness. Volume I presents an overview of the project and of the UPCAST software. Volume I describes extensions of the work beyond development of UPCAST and includes papers published to document the progress of the project. Volume II is a manual for users of the UPCAST software. Distribution of Volume II is limited to DOD and DOD contractors only.				
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS		21. ABSTRACT SECURITY CLASSIFICATION Unclassified		
22a. NAME OF RESPONSIBLE INDIVIDUAL Edward Borto		22b. TELEPHONE (Include Area Code) (313) 574-8711	22c. OFFICE SYMBOL AMSTA-TMM	

SECURITY CLASSIFICATION OF THIS PAGE

SECURITY CLASSIFICATION OF THIS PAGE

## FOREWORD

The work reported herein is the result of the effort of the personnel in several organizations. Members of the project staffs at the University of Pittsburgh, Battelle Columbus Laboratories, Blaw-Knox Foundry and Mill Machinery Company in East Chicago, Indiana, and Lebanon Steel Foundry are listed below. The program was sponsored by the U. S. Army, Tank and Automotive Command (TACOM), Warren, Michigan under Contract No. DAAK30-78C-0107. Mssrs. Thomas Wassel, Michael Holly, and Edward Borto have served ably as Project Monitors.

In addition to the project staff important contributions and advice have been given by Dr. R. Smelser, ALCOA Technical Center, formerly with the University of Pittsburgh, James Echlin, Blaw-Knox, and A. Roulet, General Dynamics.

Computing facilities on the DEC 10 system were made available by the University of Pittsburgh Computer Center. Specialized computing facilities were made available through Professors T. W. Sze (Electrical Engineering), R. Flynn (Library and Information Sciences), and R. Hoelzeman (Electrical Engineering).

The project staff at the University of Pittsburgh included: Professors H. D. Brody, R. A. Stoehr, and T. W. Sze; Graduate Student Researchers V. Srinath, W. S. Hwang, W. Schwartz, D. Shu, T. T. Wang, P. Wisniewski, S. Rao; Mssrs. J. Gasper, J. Farinelli, C. Van Ormer, and Ms. J. Frizza. The project staff at Battelle Columbus Labs included: Drs. N. Akgerman, A. Badawy, C. Wilson, and T. Altan. The project staff at Blaw-Knox included: Mssrs. R. Nariman, K. Fahey, and S. Miller. Mr. W. Northey coordinated the project work at Lebanon Steel foundry.

## TABLE OF CONTENTS

<b>1. INTRODUCTION</b>	<b>9</b>
1.1 PROJECT ORGANIZATION	10
1.2 REPORT ORGANIZATION	10
<b>2. CONCLUSIONS</b>	<b>12</b>
<b>3. RECOMMENDATIONS</b>	<b>14</b>
<b>4. GENERAL APPROACH</b>	<b>15</b>
<b>5. OVERVIEW OF UPCAST</b>	<b>19</b>
5.1 USE OF MOLD CAVITY DESIGN ROUTINES	22
5.2 USE OF ANALYSIS ROUTINES	24
5.2.1 Geometric Description and Mesh Generation	24
5.2.2 Heat Flow Simulation	25
5.2.3 Review of Heat Flow Analysis Results	26
5.2.4 Analysis of Gating Systems	27
5.3 DESCRIPTION OF MAJOR UPCAST ROUTINES	27
<b>6. EXPERIMENTAL PROGRAM</b>	<b>31</b>
6.1 STEPPED PLATE CASTING EXPERIMENTS AND SIMULATION	31
6.2 CASTING SOUNDNESS CRITERIA	39
6.3 TORSION BAR HOUSING CASTINGS	43
6.3.1 No-Bake Sand	43
6.3.2 Green Sand	44
<b>7. RESEARCH EXTENSIONS</b>	<b>48</b>
7.1 FLUID FLOW IN MOLD CAVITIES	48
7.2 INVESTMENT CASTING	50
<b>I. SIMULATIONS IN THE DESIGN OF SAND CASTINGS</b>	<b>51</b>
I.1 INTRODUCTION	52
I.2 BACKGROUND	52
I.3 CURRENT STATUS AND THE NEED FOR FUTURE DEVELOPMENT	55
I.3.1 Simplicity vs. Realistic Representation	56
I.3.2 Flexibility	60
I.3.3 Accuracy	60
I.3.4 Stability	62
I.3.5 Easily Interpreted Results	64
I.3.6 Economically Justifiable Costs	66
I.4 CONCLUSIONS	70
<b>II. A FINITE ELEMENT MESH GENERATOR FOR CASTINGS</b>	<b>73</b>
II.1 INTRODUCTION	74
II.2 GENERATION OF MESH DATA	74
II.2.1 Prepare the Geometrical Data	75
II.2.2 Input the Data to the Computer Program CSTMS3	78
<b>III. PROGRESS ON THE CAD/CAM CASTING PROCESS</b>	<b>86</b>
III.1 INTRODUCTION	87
III.2 OBJECTIVES	87

III.3 DESCRIPTION OF THE CAD/CAM CASTING PROCESS	90
III.4 EXPERIMENTAL WORK	100
III.5 FURTHER WORK	111
III.6 SUMMARY	111
<b>IV. SIMULATION OF HEAT FLOW IN CASTINGS</b>	<b>113</b>
IV.1 INTRODUCTION	114
IV.2 GENERAL HEAT CONDUCTION EQUATIONS	116
IV.3 FDM FOR PULSED WELDING	117
IV.4 FDM FOR ROLL CASTINGS	125
IV.5 FEM FOR CONTINUOUS CASTING	129
IV.6 FEM ANALYSIS OF SAND CASTINGS	134
IV.7 CONCLUSION	136
<b>V. ANALYSES OF SAND AND INVESTMENT CASTING</b>	<b>141</b>
V.1 INTRODUCTION	142
V.2 HEAT FLOW	144
V.2.1 Numerical Analysis Methods	144
V.2.2 Application to Casting Process Design	146
V.3 FLUID FLOW	151
V.3.1 Marker and Cell Technique	151
V.3.2 Filling of a Vertical Plate	152
V.3.3 Effect of Filling Rate	152
V.3.4 Filling of Plate A	152
V.4 INVESTMENT CASTING	158
V.5 SUMMARY	159
<b>VI. FLUID FLOW MODELING FOR CAD OF CASTINGS</b>	<b>161</b>
VI.1 INTRODUCTION	162
VI.2 USES OF THE BERNOULLI EQUATION APPROACH	163
VI.3 THE SAINT-VENANT EQUATION APPROACH	171
VI.4 THE MARKER-AND-CELL TECHNIQUE	174
<b>VII. METAL FLOW WITH FREE SURFACE INTO MOLDS.</b>	<b>185</b>
VII.1 COMPUTATIONAL TECHNIQUES	186
VII.2 TEST PROBLEMS AND RESULTS	190
VII.3 DISCUSSION AND CONCLUSIONS	198

## LIST OF FIGURES

<b>Figure 5-1:</b>	Overview of UPCAST.	20
<b>Figure 5-2:</b>	Major subprograms in UPCAST.	21
<b>Figure 5-3:</b>	Graphics terminal and digitizing table used with DEC 10.	23
<b>Figure 5-4:</b>	Computed filling sequence of a stepped plate casting.	28
<b>Figure 6-1:</b>	Side view of Blaw-Knox pattern for test plates.	32
<b>Figure 6-2:</b>	Positions of 16 thermocouples in plate castings.	32
<b>Figure 6-3:</b>	Instrumented plate castings at Blaw-Knox foundry.	35
<b>Figure 6-4:</b>	Computed (dash) and measured (solid) cooling curves, plate A.	38
<b>Figure 6-5:</b>	Isothermal contours predicted for plate casting A.	40
<b>Figure 6-6:</b>	Temperature gradients in freezing range, computed.	41
<b>Figure 6-7:</b>	Temperature gradients in freezing range, measured.	42
<b>Figure 6-8:</b>	Drag pattern & mold for configuration I.	45
<b>Figure 6-9:</b>	Cope pattern (one set) & mold for configuration I.	46
<b>Figure 6-10:</b>	Rough castings (configuration I) right after shakeout.	47
<b>Figure 7-1:</b>	Computed filling pattern for plate casting A.	49
<b>Figure 1-1:</b>	Typical sand casting is a complex assemblage.	53
<b>Figure 1-2:</b>	The casting and rigging must be divided into a mesh.	57
<b>Figure 1-3:</b>	Large steel roll castings were modeled to predict bridging.	58
<b>Figure 1-4:</b>	Two ways of representing the heat capacity and latent heat.	61
<b>Figure 1-5:</b>	Solutions for convective cooling of a fin.	63
<b>Figure 1-6:</b>	Plotter representation of solidifying plate casting.	65
<b>Figure 1-7:</b>	Plots of cooling rates along centerline of a plate casting.	67
<b>Figure 1-8:</b>	Thermal gradients along the centerline of a plate casting.	68
<b>Figure 1-9:</b>	Line printer representation of roll solidification.	69
<b>Figure II-1:</b>	Test casting model.	76
<b>Figure II-2:</b>	Division of test casting model into zones.	77
<b>Figure II-3:</b>	CRT representation of zone model.	81
<b>Figure II-4:</b>	Scaling and rotation of the zone model.	83
<b>Figure II-5:</b>	3-D plot for layer number 3.	85
<b>Figure III-1:</b>	Overall design of the CAD/CAM casting process.	89
<b>Figure III-2:</b>	Example of cross-section data input.	91
<b>Figure III-3:</b>	Drag half of stepped plate casting model used for mesh generation.	93
<b>Figure III-4:</b>	Simulated freezing progression map for stepped plate castings.	95
<b>Figure III-5:</b>	Side view of pattern of test plate castings.	101
<b>Figure III-6:</b>	Top view of test plate castings indicating thermocouple positions.	102
<b>Figure III-7:</b>	Cooling curves for casting A at centerline positions.	105
<b>Figure III-8:</b>	Temperature distribution along centerline of A at several times.	106
<b>Figure III-9:</b>	Cooling curves for casting D at centerline positions.	107
<b>Figure III-10:</b>	Temperature distribution along centerline of D at several	108

	times.	
<b>Figure III-11:</b>	Thermal gradients along centerline in freezing range for A and D.	109
<b>Figure IV-1:</b>	Example of element mesh suitable for pulsed arc welding.	118
<b>Figure IV-2:</b>	Definition of terms used in pulsed arc welding.	119
<b>Figure IV-3:</b>	Flow chart for finite difference analysis of pulsed GTAW.	120
<b>Figure IV-4:</b>	Thermal profile at change from peak to background power.	123
<b>Figure IV-5:</b>	Computed temperature variation in pulsed arc welding.	124
<b>Figure IV-6:</b>	Large steel roll used for simulation. Time: 4 hours.	126
<b>Figure IV-7:</b>	Large steel roll used for simulation. Time: 8.5 hours.	128
<b>Figure IV-8:</b>	Triangular ring elements used in FEM for DC casting.	130
<b>Figure IV-9:</b>	Liquidus and solidus isotherms, 38 cm $\phi$ , 10 cm/min.	132
<b>Figure IV-10:</b>	Procedure for computer aided design & simulation of castings.	135
<b>Figure V-1:</b>	Configurations of stepped plate test castings.	143
<b>Figure V-2:</b>	Cooling curves, Plate A. (a) 2-D simulation (dashed). (b) Measured (solid).	147
<b>Figure V-3:</b>	Cooling curves, Plate A. (a) 3-D simulation (dashed). (b) Measured (solid).	148
<b>Figure V-4:</b>	Thermal gradient during freezing, centerline, Plates A & D.	150
<b>Figure V-5:</b>	Velocity vectors computed for filling of a vertical plate.	153
<b>Figure V-6:</b>	Effect of filling rate on filling of a square mold cavity	156
<b>Figure V-7:</b>	Velocity vectors, simulated, for filling of plate A.	157
<b>Figure VI-1:</b>	Factors in the determination of the filling time of a mold.	164
<b>Figure VI-2:</b>	Sprue taper design to avoid free fall turbulence.	166
<b>Figure VI-3:</b>	Equal flow through multiple runners from a common sprue.	168
<b>Figure VI-4:</b>	Unequal flow through multiple gates from a single runner.	169
<b>Figure VI-5:</b>	Flow distribution in gates shown in Figure VI-4.	172
<b>Figure VI-6:</b>	Saint-Venant representation of flow along a sloping runner.	173
<b>Figure VI-7:</b>	Block flow chart of the MAC calculation scheme.	177
<b>Figure VI-8:</b>	Vector plots (MAC) of flow into horizontal square molds.	178
<b>Figure VI-9:</b>	Vector plots of flow into mold with vertical steps on drag.	180
<b>Figure VII-1:</b>	Flow chart of FPMAC program.	188
<b>Figure VII-2:</b>	Marker plots of metal entering horizontal square cavity.	191
<b>Figure VII-3:</b>	Vector plots of metal entering square cavities.	192
<b>Figure VII-4:</b>	Vector plots of flow into vertical mold, with SMAC.	194
<b>Figure VII-5:</b>	High-speed movie frames of flow into a vertical mold.	195
<b>Figure VII-6:</b>	Vector plots, entry to stepped mold cavity, with SMAC.	196
<b>Figure VII-7:</b>	Vector plot for complete casting system, with SMAC.	197

## LIST OF TABLES

<b>Table 6-1:</b>	Dimensions of Test Plate Castings A - H.	33
<b>Table 6-2:</b>	Chemical Analysis of Stepped Plate Castings.	34
<b>Table 6-3:</b>	Thermal Properties of Steel & Sand for Simulation.	36
<b>Table III-1:</b>	Dimensions of Test Plate Castings.	103
<b>Table III-2:</b>	Chemical Analyses (wt %) of Test Plate Castings.	104
<b>Table IV-1:</b>	Effect of Secondary Cooling on Heat Flux in D C Casting.	133
<b>Table VII-1:</b>	Relation of FORTRAN and Algebraic Symbols.	189

## 1. INTRODUCTION

A computer assisted methodology for the design and analysis of the casting process for manufacturing sound, high integrity steel castings in sand molds has been developed. The design and analysis routines, which are linked by an overall supervisory program, UPCAST, offer foundrymen the ability to interactively use computer graphics and specially tailored analysis packages to design the mold configuration for a steel casting and to analyze the filling and freezing patterns of the casting. Through finite difference and finite element techniques in the UPCAST software, mold designs (i.e., the rigging of castings) may be updated based on the results of heat flow and fluid flow simulations. In parallel with the development of the software for UPCAST, an experimental program was undertaken to do the following:

- Provide the data base necessary to successfully implement the analysis routines;
- Validate the models and computer programs;
- Develop casting soundness criteria suitable for computer assisted design of the casting process for low alloy steels (as used in ground based armor) cast in sand molds.

The routines and data in UPCAST have been developed for casting steel in sand molds; however, the routines are general and may be adapted for other alloys, mold materials, and casting processes. The assumptions made in developing the analysis routines are listed within this report. The basis of the simulation routines for computing freezing patterns should be applicable to sand casting of any alloy and to any casting process in which the molten alloy is poured into thick, insulating molds. The input data for different alloy/mold combinations may be obtained by using the techniques described in this report for steel/no-bake sand and steel/green sand. All simulation routines, except for completely automatic mesh generation (MGEN), may be extended by an experienced user to the analysis of casting in conductive (metal) molds, in thin (shell and investment) molds, and in composite molds. The research work on fluid flow analysis and thermal behavior of investment material reported here is intended to extend the general applicability of computer design and analysis procedures to castings.

Only very general computations and graphical analyses have been used in the design routines. These design procedures are consistent with commonly sited methods of riser size and placement determination. If a particular user has favorite

design methods (risering or gating calculation procedures), they may be incorporated into the overall UPCAST framework. Data for particular alloy/mold combinations and on commonly used geometries in a particular foundry/cast shop may be built into the program.

## 1.1 PROJECT ORGANIZATION

The work was supported by the U. S. Army Tank - Automotive Command (TACOM) in Warren, Michigan. The University of Pittsburgh, as the prime contractor, monitored the work on all phases and tasks of the project and was involved in all technical aspects of software development and experimentation. There were three subcontractors.

Battelle Columbus Laboratories contributed substantially to the development and modification of geometric modeling and computer graphics routines for the description and design of mold configurations. The special expertise in computer graphics developed at Battelle for the computer aided design and computer aided manufacture (CAD/CAM) of forging processes served as a starting point for many of the computer graphics routines developed under this project and used in the UPCAST software to assist in the analysis and design of the casting manufacturing process.

Blaw-Knox Foundry and Mill Machinery Co. cast several instrumented test plate castings and over 100 demonstration torsion bar housing castings, of which several were instrumented. All were cast in no-bake (organically bonded) sand molds. Lebanon Steel Foundry cast four instrumented test plate castings and ten demonstration torsion bar housings, 2 with instrumentation, using green (clay bonded) sand molds. Personnel from both foundries made valuable suggestions relative to directing software development with practical useful results.

## 1.2 REPORT ORGANIZATION

This segment of the report, Volume I, lists the major tasks undertaken during the project and summarizes the results of each task. The general approach of the project to the development of a software package for the design and analysis of steel casting and an overview of the UPCAST package are presented. Also included are summaries of the experimental and research phases of the work. Of particular importance are the descriptions of the experiments to obtain thermal data on the steel/sand combination considered here, on the extension of fluid flow analyses to the open channel flow of molten alloys into complex mold cavities, and on

determining the thermal behavior of investment shell materials. Papers that give more details on important aspects of the project, particularly the research extensions, have been appended as exhibits.

Volume II is a manual for users of UPCAST. It describes the function of each of the major software routines within UPCAST and gives detailed instructions on the use of each. Several examples of the use of individual routines on casting processes are given. Volume II may be used as a guide by a foundryman/casting engineer who wishes to use any one or group of UPCAST routines. Additionally, a software engineer or materials processing engineer may use the program descriptions and program design philosophies described in Volume II as a guide and framework for writing new casting analysis and design routines. The new routines would take advantage of advanced criteria for predicting casting *soundness* and cast properties and technical advances in computer hardware and software.

## 2. CONCLUSIONS

UPCAST, a comprehensive software package, has been developed for the analysis and design of the solidification process in alloy castings. UPCAST incorporates numerical simulation, geometric modeling, and computer graphics with advanced solidification principles, practical design techniques, and casting soundness criteria within a unified framework. UPCAST is designed to expand the analysis capabilities of foundrymen and casting engineers by providing easy control of powerful computer routines and ready interpretation of computed results in terms of parameters that influence casting soundness. All decisions on casting practice are reserved for the foundrymen.

- The UPCAST routines, when combined with appropriate data bases for material properties and solidification behavior, may be used as presently developed for the analysis of static casting processes; including, sand casting, investment casting, permanent mold casting, die casting, and ingotmaking. However, UPCAST will best serve as a model for computer aided analysis and design routines.
- The software framework is modular and improved routines, as they are developed, may be substituted easily for existing routines. UPCAST has been implemented on two hardware systems. The modular design and the use wherever possible of generally available programming languages will make conversion of UPCAST, especially the analysis routines, to new hardware systems relatively easy.
- The solidification analysis routines, 2-D and 3-D, finite element and finite difference numerical simulations, are state of the art. They were based on the best routines available at the time the project was initiated and they have been continually upgraded.
- The geometric modeling and computer graphics techniques and programs that were available at the time this project was initiated were limited in comparison to present software capabilities. The routines in UPCAST will serve as a model. Serious users of CAD/CAM for casting will incorporate up to date hardware and software.
- Interfacing of geometrical modeling with numerical simulation is a critical step in providing packages that can find general use by foundrymen. Discretization of a complex geometry into a mesh of elements and nodes for simulation by a finite numerical technique is generally tedious and challenging. This program has made a first step in developing techniques for automatic and semiautomatic mesh generation using logic suitable for the simulation of solidification.

Thermophysical properties suitable for the analysis of the casting of low alloy steel into sand molds have been obtained by comparing the results of computer analyses with thermal measurements of laboratory and foundry castings. Considering the normal range of variation in commercial steel foundry practice, several simplifications may be made in the representation of materials properties.

- Temperature independent properties may be used except in the case of the representation of enthalpy within the freezing range.
- The rate of enthalpy release within the freezing range is best represented by two or three values divided, for example, from 0-60% solid, 60-90% solid, and 90-100% solid.
- The same properties may be used to represent clay bonded green sand and organically bonded no-bake sand.

Normal casting design procedures may be used with computer programs, however, the real potential of computer aided analyses rests in developing design procedures and criteria that make use of the advanced analyses based on the capabilities of machine computation.

- Progressiveness of the advance of (computed) isotherms in the freezing range may be used to predict areas of macroshrinkage.
- Temperature gradients during freezing (computed) may be compared with a critical value as a criterion to predict centerline porosity and microshrinkage.
- Small and medium sized steel castings poured in sand molds will not be radiographically sound in regions where the temperature gradient during freezing is less than one degree Farenheit per inch.

Simple routines for the analysis of fluid flow and heat loss during mold filling were incorporated in UPCAST. In simplest use, UPCAST routines assume metal at an initial uniform temperature enters the mold which is also at a uniform initial temperature. Thus, the influence of pouring practice on solidification pattern and soundness are not taken into account. This would be a major limitation, in particular, for the analysis of thin walled castings.

Work initiated under the project to develop much more powerful routines for the analysis of the open channel transient flow during mold filling indicates the potential for much improved analytic capabilities.

### 3. RECOMMENDATIONS

Foundries supplying advanced castings, should be encouraged to implement computer aided analysis and design of the casting process. UPCAST may be used as a model system. UPCAST analysis routines are portable and state of the art. Foundries should select graphics techniques compatible with internal needs.

Data bases of thermophysical properties should be developed for casting alloys and mold materials important for defense needs.

More progress is required on automatic mesh generation for casting analysis before widespread use of solidification analysis by computer will be adapted widely by foundrymen.

The real potential for advancing casting practice through computer aided analysis and design rests in the development of new criteria to predict casting soundness and as-cast properties. Research in this area should be given priority.

Development of fluid flow models of mold filling should be continued and the flow analyses combined with heat transfer and solidification analyses. This will be practically important for the analysis of thin walled castings, die casting, and investment casting.

#### 4. GENERAL APPROACH

A unique feature of this work is the integration into a single package of routines for both the analysis and design of casting processes. The software has been designed with several criteria in mind. Of particular importance was the objective of making UPCAST a tool that would give foundrymen/casting engineers the opportunity to use computer graphics in their normal decision making processes and to afford the users some extra, more powerful design and analysis options not normally at their disposal.

It was important in this framework to retain all decisions relative to casting as the responsibility of the foundryman. On the other hand, to make the new approach amenable to practical use in many foundries/cast shops, the routines must be simple and menu driven with commands easily remembered and associated directly with their function. Decisions relative to the method of computation and application of numerical techniques are retained by the software. However, provision is made for the experienced user to take more control of the analysis routines and to override or avoid automatic features.

Another important criterion has been to maximize portability of the software. In that respect, FORTRAN was used to code software to the maximum extent possible. Nonetheless, some routines required a specialized coding language. In the case of plotting and graphics display routines, the widely available CALCOMP and PLOT10 languages, respectively, were used in the version of the UPCAST software documented in Volume II: UPCAST Users Manual.

Options were maintained for the use of UPCAST with minimal investment in computer hardware. One approach was to use UPCAST on a timesharing service. In such a case, the foundry would have to invest, as a minimum, in a terminal capable of graphics display and a modem for telephone linkage to a timesharing system. A digitizing tablet would be a valuable, inexpensive adjunct to a casting process design system.

The next category of capital outlay would be for a minicomputer, such as one from the PDP11 family, to give the foundry/cast shop the capabilities for graphics design and preprocessing and postprocessing of the analysis routines in house. The foundry would still maintain a service arrangement on a mainframe to process (as batch jobs) the finite element or finite difference solidification simulation routines. Additional options would depend upon existing equipment or anticipated commitments of the foundry.

The analysis routines using numerical simulation have been kept at the level of sophistication that, at initiation of this project, would require a mainframe computer. Simple design routines based on normal foundry rules of thumb and nomographs can easily be written into the design routines of the graphics software. However, to offer the foundryman more powerful analysis tools than otherwise available, it was deemed important to offer simulation routines based on the finite difference and finite element numerical methods. These methods require fast calculation and extensive storage, preferably core storage, capabilities in the computer system used. The mainframes required are not usually available, on a dedicated basis, to most foundrymen. However, they are accessible on a timesharing basis. On the other hand, it has been our assumption that the speed and capacity of computers of all types would continually be expanded as their costs were decreased. Thus, computation schemes that were not practical at the beginning of the project would become closer to the capabilities of a larger portion of the foundries as the project work went on. Indeed, this has been the case. Minicomputers are available and affordable to handle calculations on the scale that we initially had available on a mainframe; and the capabilities of mainframes have expanded tenfold. We expect this trend to continue. Finite difference and finite element analysis should be a realistic endeavor for the majority of foundries and cast shops.

- It should be kept in mind that while a foundry may decide to purchase a slow computer system relative to engineering calculations, that computer system will be dedicated to the foundry's use. At night, when normal process control and accounting functions do not require computer time, the computer can be left alone to carry out the computations for a casting analysis. Results of analyses would be available for review by the foundryman the next morning.

The procedure of carrying out the finite element and finite difference analyses of heat transfer in castings can be made completely automatic (transparent) to the user. All decisions relative to the numerical computation and its implementation on the computer can be programmed into the software so that a user need only request the analysis be done. However, the preparation of the information needed for the simulation and the review of the results of the simulation require user interaction.

In UPCAST, review of the voluminous data that result from the simulations is performed graphically and in terms of parameters and criteria important to casting analysis. The user merely has to select from a menu the type of graphical analysis desired, enter the name of the file containing the data, and indicate the region in which the analysis is requested.

Preparing the data files that will describe to the analysis programs the geometry of the casting and mold and the nature of the simulation that is to be handled requires more user interaction. In particular, the generation of a finite element or finite difference mesh is an extremely important, but tedious task requiring experience with numerical computations on the part of the user. In UPCAST, an attempt was made to supply an option that would make the preparation of finite element meshes automatic. A simple logic, applicable to the analysis of heat transfer in castings, was built into the automatic 3-D mesh generation routine, MGEN. This routine can handle shapes that represent a wide variety of casting configurations, but not all castings. Further, a knowledgeable user will be able to create a more efficient mesh than the one created by MGEN. Yet, this is a start toward bringing the potential of sophisticated numerical techniques to a class of users who have no experience or detailed understanding of their operation.

The development of the UPCAST software, described in detail in Volume II of this report, comprised the major effort on software development. Additionally, several experimental, research, and computational tasks were important aspects of this work:

- Support the development of the method for computer assisted analysis and design of the manufacturing process for high integrity castings used in the present version of UPCAST.
- Extend the current understanding and data base so that the range of applicability of computer assisted manufacturing software, such as UPCAST, may be extended to casting processes and ranges not presently practical.

In line with these objectives tasks were proposed and carried out with the following objectives:

- Determine the approach to be used in writing the UPCAST software for its implementation in foundries and cast shops;
- Develop a data base of material properties suitable for the computer simulation of the freezing pattern of low alloy medium carbon steel in sand molds;
- Test the validity of the computer software by measuring the behavior of castings produced in commercial foundries under typical production conditions in order to check and correlate the measured behavior with the results of computer simulations;
- Develop criteria suitable for use with computer analyses that may be used to adequately predict the soundness of castings;

- Test the applicability of the model to castings of complex geometry processed in complex mold configurations;
- Extend the analysis of the flow of molten alloys during casting to include the simulation of transient flow of the alloys into the free surface configuration of a mold cavity;
- Extend the software to be able to handle adequately the design and analysis of the investment casting process.

As the project progressed, papers were prepared (a) for presentation at national meetings sponsored by professional societies and by the Department of Defense and (b) for publication in appropriate journals and proceedings to disseminate the ideas and knowledge of the software being developed under this contract to those who could expand their adoption. The papers contain descriptions of the experimental and research phases of this project and are appended as exhibits to this report both as a record of the documentation already prepared and to serve as a source of details. A few additional papers on this and related work will be prepared at a later date.

Two papers presented at the first symposium on "Modeling of Casting and Welding Processes" relate to the approach taken in developing the software. The first paper, included as Exhibit I describes the overall approach to casting analyses. The second paper, included as Exhibit II, describes the approach to automatic mesh generation.

## 5. OVERVIEW OF UPCAST

UPCAST is a collection of software routines to assist foundrymen in making decisions on the process of manufacturing sound, high integrity sand molds. The program allows interactive processing. The user enters simple commands and data via a keyboard or digitizer. Questions and guidance on the use of the programs are generated by the software on the user's terminal (teletype or C.R.T.). It is intended that UPCAST will be usable by foundrymen and casting specialists who are not experts either in computer techniques or the sophisticated methods of numerical analysis of heat transfer and fluid flow. These tools are made available within UPCAST to assist in the decision making process, yet the foundryman retains and makes all decisions relative to the casting process. Provisions are made that a user with a good background in computer techniques and numerical methods may override the automatic provisions of the software and guide the analysis more directly.

An overall representation of the functions of UPCAST is presented in Figure 5-1. A diagram showing the major programs in UPCAST is presented in Figure 5-2. UPCAST is an integrated set of programs aimed at providing a computer aided analysis and design tool for casting engineers in the two major areas:

- The design of the mold configuration with the aid of computer graphics and geometric modeling. A framework has been developed for utilizing computer graphics to handle and display cross-sectional geometry as well as calculate geometric parameters relative to heading (risering) and gating. Standard design procedures are used.
- The analysis of the freezing and filling pattern of a steel alloy in a given mold configuration using advanced numerical simulation techniques. Finite element and finite difference analyses of the solidification process are used for evaluated mold cavity design. Cross-sections entered in the design routines may be used for two-dimensional analysis of casting sections. Also, facilities are provided for defining a simplified three-dimensional mathematical model of the casting. Actual mesh generation for the analysis may be completely automatic. Output in the form of heat transfer plots and solidification maps will be used with casting soundness criteria to predict casting integrity.

The programs are tied together by an overall supervisory program. Individual programs may be called in any order by entering UPCAST and typing the commands which are underlined in each program box in Figure 5-2 and listed below.

The routines may be used in sequence (i.e., use UPCAST for design of a mold configuration and then use UPCAST for analysis of the developed design). Or mold

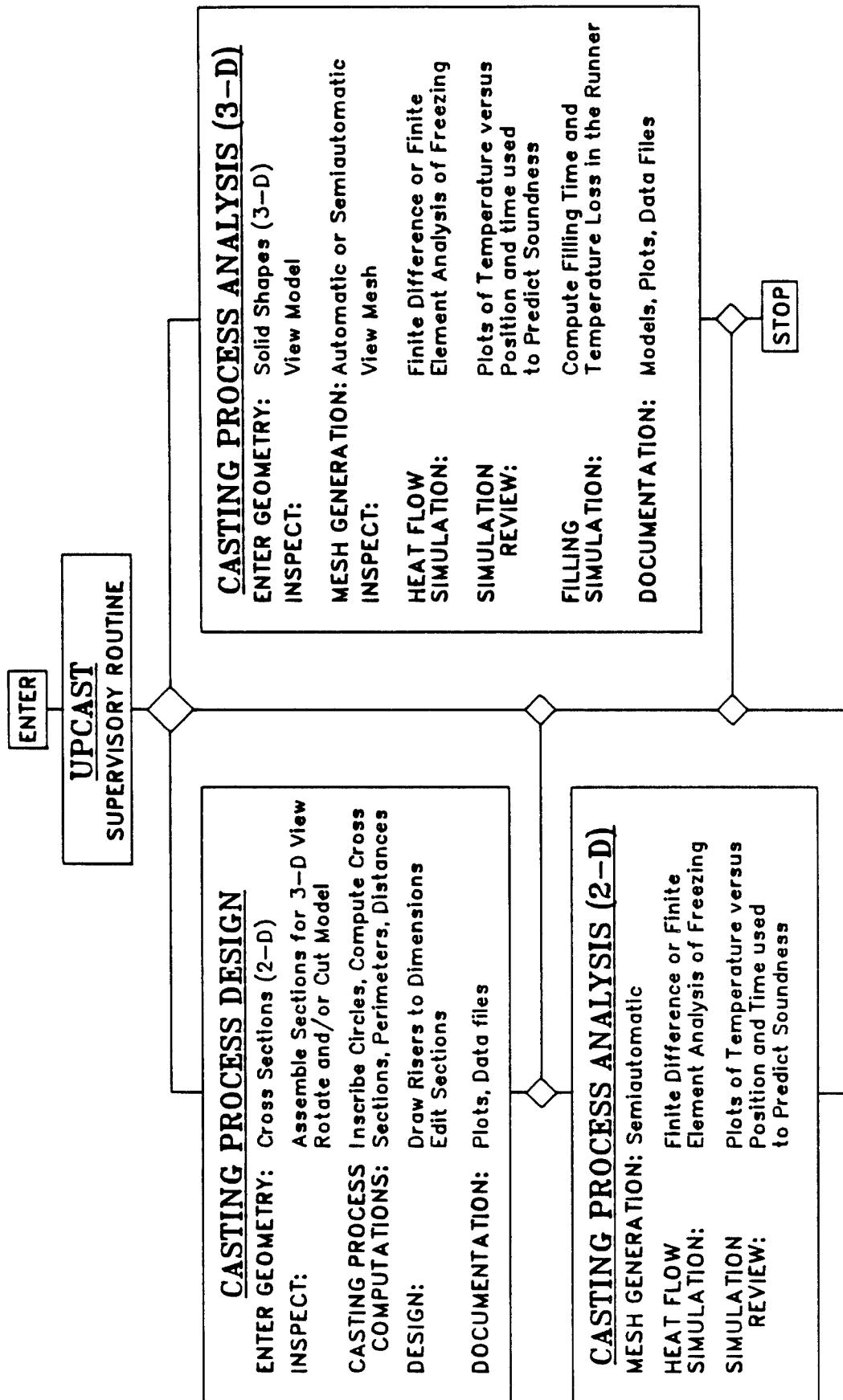


Figure 5-1: Overview of UPCAST.

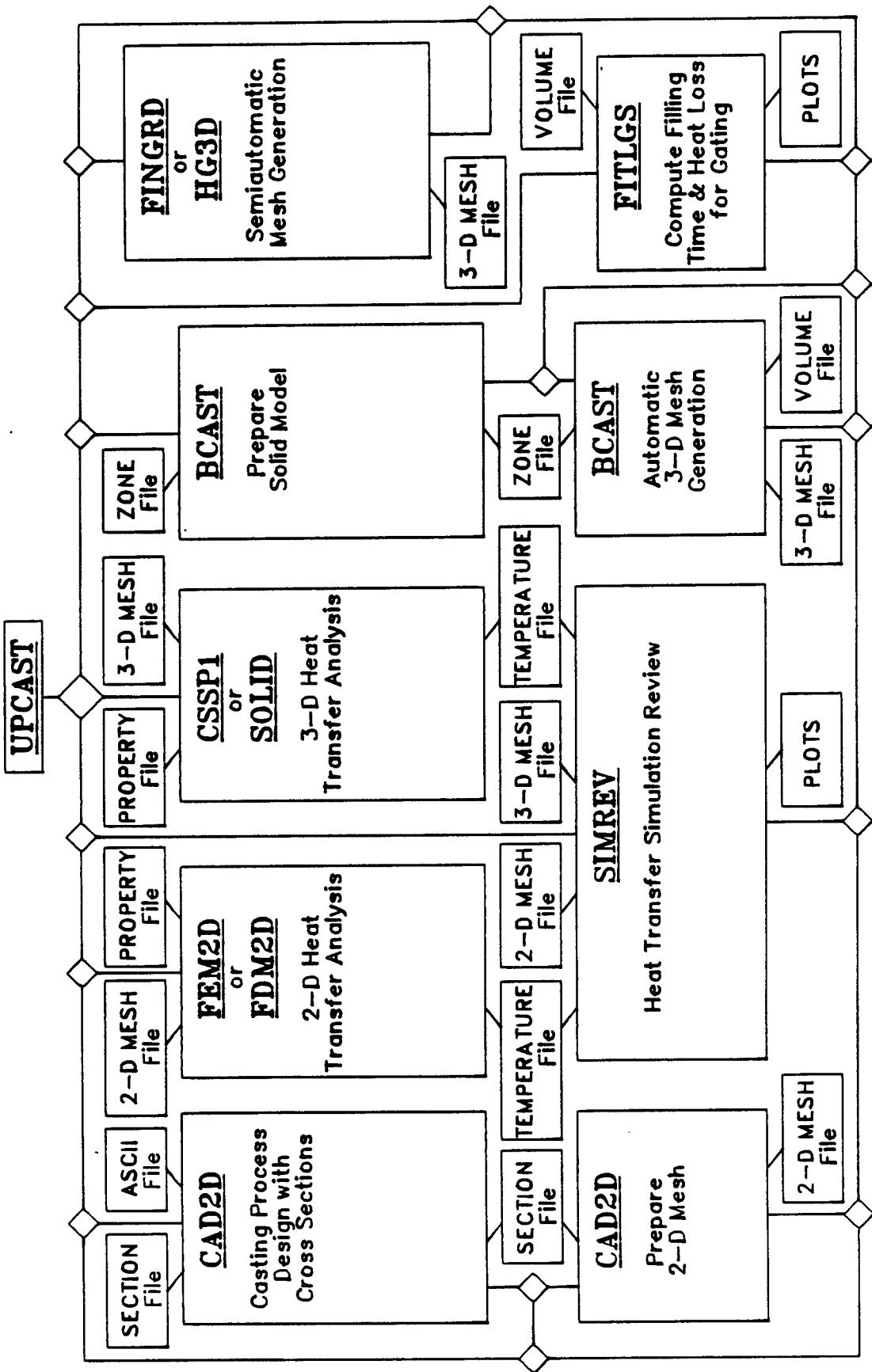


Figure 5-2: Major subprograms in UPCAST.

layout may be planned with the computer graphics design routines and numerical analysis omitted. Or the design may be done by another means and the design checked by numerical simulation of the freezing or filling pattern using UPCAST.

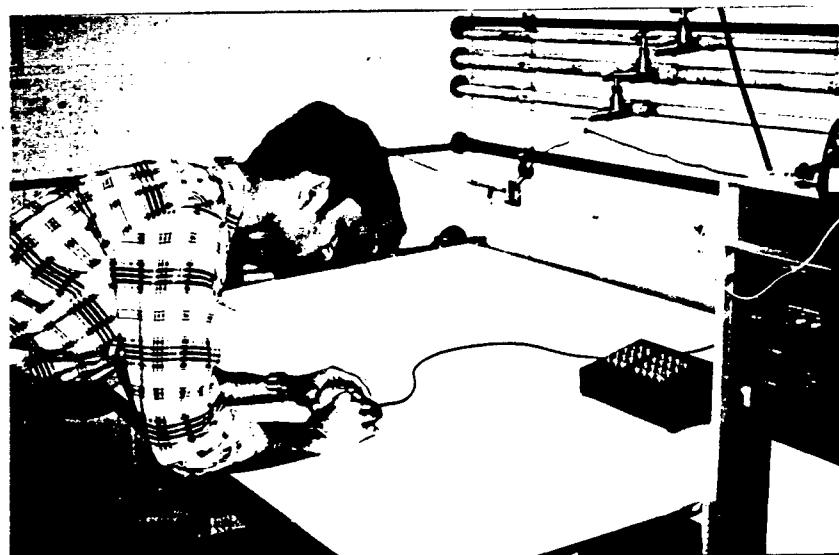
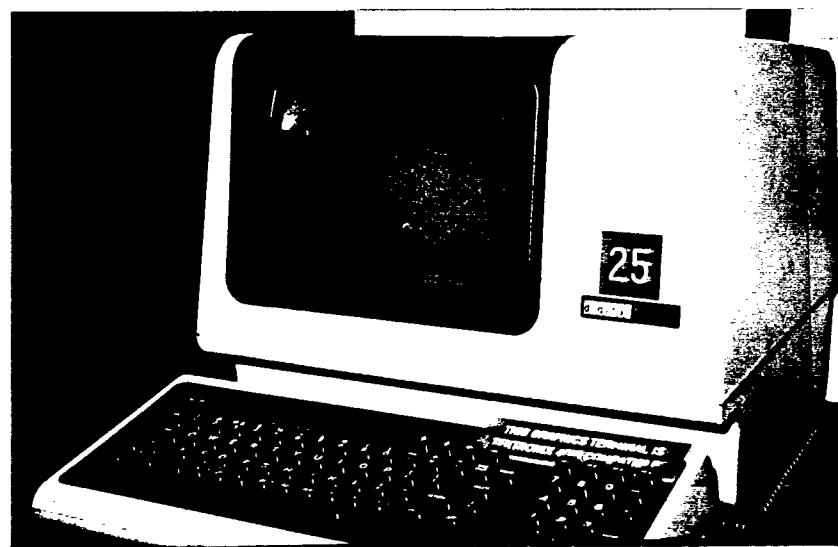
The UPCAST software is driven by a hierarchy of command menus. Once UPCAST is entered, any of the major programs may be called by giving a simple command in response to a query from the program. When a program is entered from UPCAST, the functions to be performed by routines within the program are specified by typing simple commands from the highest level menu within the program. If further specification of function is required, the user responds to queries from the program by selecting commands from lower level menus. Generally the lower level menu selections will be listed on the screen. The major menus may be listed by typing HELP in response to a query or by typing an incorrect command. Details are given in the UPCAST User's Manual, Volume II.

Communication of information between different programs is done through preparing data files in preset formats. For example, the outline of the cross section of a casting may be described and entered into memory using CAD2D. The description of the section will be stored in a section file. CAD2D may then be used to create a 2-D finite element mesh of the cross section stored in the section file with the result stored in a 2-D mesh file. Later the mesh file may be used as input to CSSP2D for heat transfer analysis. The results will be stored in a temperature file (temperature at each node in the mesh computed at several timesteps from the time the mold was filled) suitable for input along with the mesh file to SIMREV where graphical analysis of the results of the computations may be made.

## 5.1 USE OF MOLD CAVITY DESIGN ROUTINES

The mold cavity design routines are primarily a drafting tool that may make use of several input and output devices. Figure 5-3 illustrates (top) a graphics screen that may be used with a *light pen* and keyboard and (bottom) a digitizing table for input of geometric data. With the keyboard, coordinates of points describing the perimeter of a cross section can be input. Twenty-five points per cross-section can be used. With the use of different function keys, the points can be joined by straight lines, arcs, or smooth curves. The cross-section may be viewed as it is input on the graphics screen.

With the digitizing table (or tablet), cross-sections can be input by entering with a cursor button coordinates of points from a print placed on the table. Sections or



**Figure 5-3:** Graphics terminal and digitizing table used with DEC 10.

views of a scaled print can, in that way, be *traced* onto the screen and into a data file. Again, several function keys aid in this process. For example, a circle can be *traced* by digitizing the center and one point on the circumference, an arc of a circle by digitizing three points, and a smooth curve by digitizing several points. The sections on the screen can be enlarged or reduced to any desired scale and shrinkage allowances can be added.

When all of the cross-sections defining a cavity have been fed to the computer, the computer can be ordered to display a three-dimensional, isometric view of the casting. The 3-D view may be rotated and seen from different angles.

At this time the casting engineer/foundryman is ready to interact with the system by adding machining allowances, back-ups, fillers, etc. using the graphics terminal, light pen, and keyboard. The program can be ordered to inscribe circles along a cross-section and to display the diameters of these circles. It can also calculate and display selected distances, surface areas, and perimeters.

Then, the casting engineer may locate the calculated feeder pads, risers, chills, gating, etc. with the aid of the input devices. Hard copies of the intermediate and final designs may be printed automatically.

Descriptions of the major design routine (CAD2D) and its functions are presented in a later section and in detail in the UPCAST Users Manual, Volume II.

## 5.2 USE OF ANALYSIS ROUTINES

### 5.2.1 Geometric Description and Mesh Generation

To evaluate a mold cavity design, such as a design developed using UPCAST, routines are available to perform simple fluid flow calculations and sophisticated 2-D and 3-D heat flow simulations. As a first step the geometry of the mold cavity and flask must be described. The sections developed for a design session may be used as the starting point for a 2-D analysis. CAD2D has been given the capability to assist the user in easy generation of a 2-D mesh. Files are created which are compatible with the 2-D heat transfer analysis routines.

Three-dimensional (solid) description of the mold cavity and flask geometry is entered in terms of simple geometrical shapes called zones. The zones are then divided into nodes and elements creating data input files for the 3-D analysis routines. Three routines are available for entering simplified solid models and for

mesh generation. BCAST was developed for virtually automatic mesh generation. GMSH3D and MG3D were developed for less automatic mesh generation for finite element and finite difference analyses, respectively. In general, programs requiring more user interaction offer the user more flexibility in the use of the program.

For BCAST the geometry of the mold cavity, including risers and casting, and of the mold sand, chills, insulators, sleeves, etc. are described to the computer in terms of zones using the graphics screen and keyboard interactively. The shapes implemented include plates (bricks), cylinders, and cones. Hollow cylinders and cones may be entered with certain restrictions. A significant objective for BCAST has been that, opposed to other existing analysis routines, the nodal mesh for finite element analysis may be generated automatically with little guidance by the user.

All three mesh generation routines distinguish between nodes located in metal, sand, etc.; identifies the type of element generated, e.g., brick or wedge; numbers each node and lists the X, Y, and Z coordinates of each node. For finite difference analyses neighboring nodes must be identified. Also specified are boundary and interface conditions of heat transfer. Of particular note is a translator to convert a finite element mesh file as created by UPCAST to a finite difference mesh file for use with the heat transfer routine SOLID.

### 5.2.2 Heat Flow Simulation

A significant objective of the heat flow evaluation routines has been to allow both two-dimensional and three-dimensional heat flow analyses. Two dimensional analyses of critical sections are economical in terms of computer (and user) time, storage, and analysis. three-dimensional analyses will be useful for complex castings, especially where lateral feeding is important.

The finite element and finite difference heat transfer routines, CSSP2D, CSSP3D, and SOLID, are designed to simulate arbitrary casting geometries considering conduction in the alloy and sand and convective loss to the ambient. It is assumed that risers and casting are filled instantaneously, and then cooling is simulated. Preheat of mold materials and superheat of the alloy input for the simulation need not be uniform. Latent heat of the alloy is taken into account by modifying the values of heat capacity used for the alloy within the freezing range. The thermal and physical properties of all materials are included in a data file accessed by the program. The data included with UPCAST have been selected for armor steel/no-bake sand. They may be easily altered or reproduced for other alloy/mold material combinations.

Typically, finite element analysis routines are suitable for the simulation of complex shapes and for using irregular elements in the mesh. This flexibility is built into the UPCAST FEM routines. The FDM routines are unique in their ability to handle meshes containing irregularly shaped elements.

The programs are typically submitted as batch jobs. Thus, the user's main interaction with the analysis routines is to create or select the control file specifying which mesh geometry, material, and casting conditions,etc. are to be analyzed. The programs are prepared to accept the input files that were prepared by the user with interaction with UPCAST. The program will create data files that are compatible with the post-processing review routines. The user may, but is not recommended, to analyze the data output files directly. The heat flow analysis routines may be run in timesharing mode, however, it is not expected that such operation will be practical for typically complex casting geometries with the current computer hardware capabilities.

### 5.2.3 Review of Heat Flow Analysis Results

The results of the heat flow analyses may be viewed in several forms.

1. Progress of freezing maps where the temperatures at the nodal points are indicated by numbers or special symbols. Maps may be requested on any plane for any timestep.
2. Progress of freezing contours. Isotherms may be selected at up to five key temperatures. Contour maps of the progress of freezing and cooling may be selected for any plane and for any timestep.
3. Cooling curves, temperature versus time at a given position or set of positions.
4. Temperature distribution, temperature versus position at a given time or a set of times.
5. Cooling rate at a given position plotted as a function of position at a given time or set of times.
6. thermal gradient at a given position plotted as a function of position at a given time or sets of time. Of particular value to casting soundness prediction is temperature gradient in a region at the time that region is undergoing freezing.

#### 5.2.4 Analysis of Gating Systems

Two simple calculations to simulate the behavior of a gating system have been included in the current version of UPCAST, primarily within the routine FITLGS. One analysis computes the filling time of the casting. It also computes the flow rates and energy losses in the various parts of the gating system. A second simulation that is available makes use of the the result of the flow analysis to compute the maximum temperature drop in the gating system. The latter computation is an aid in planning the tapping and pouring temperatures. Figure 5-4 is an example of the use of FITLGS to predict the filling pattern and filling time for a stepped plate casting.

The present computation uses the volumetric geometry of the mold cavity that would be input with BCAST's solid modeler. The description of the gating system, including dimensions and shapes of runners, sprues, and ingates, is entered interactively by the user in response to guidance from FITLGS.

More sophisticated fluid flow analysis schemes were started under development during this contract. They are discussed in a later chapter.

### 5.3 DESCRIPTION OF MAJOR UPCAST ROUTINES

The major programs in UPCAST are described below. Reference should be made to Figures 5-1 and 5-2. The UPCAST Users manual, Volume II, describes each routine in detail and gives examples of use.

UPCAST - The overall supervisory program that allows the user to enter each of the major UPCAST routines, perform the functions desired, and then to return to the UPCAST supervisor in order to choose another routine. (A frequent user may select to run the execute files for the UPCAST routine desired.) Some major routines are commonly run as batch jobs. In such a case UPCAST helps the user prepare a control file for submission of the simulation as a batch job. The results of the simulation would be placed on tape. The user will be able to recall the data from tape and use UPCAST routines to review the key aspects of the computed data. UPCAST calls the major programs with these commands: CAD2D, GMSH2D, CSSP2D, FDM2D, BCAST, GMSH3D, CSSP3D, SOLID, SIMREV, and FITLGS. Other UPCAST commands are HELP and END.

CAD2D - These routines are used to enter the geometry of a casting and to alter the geometry to the configuration necessary to form a pattern. The geometry is described in terms of cross sections; several cross sections may be used to build

FILLING TIME: 7.30

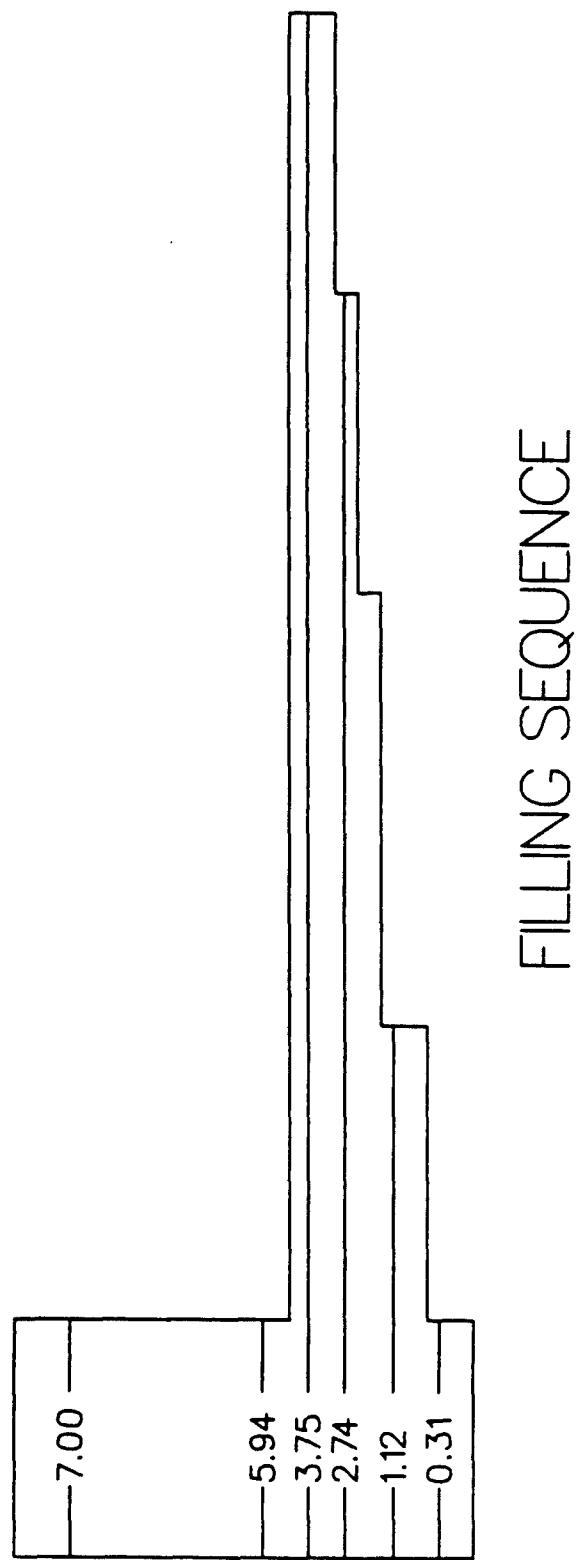


Figure 5-4: Computed filling sequence of a stepped plate casting.

the model. The cross sections may be described to the software by using the keyboard to enter the coordinates of key points; more expeditiously, the coordinates may be entered by tracing different views from a blueprint. The cross sections may be assembled on the graphics screen in proper orientation for a 3-D view and the assembled sections may be rotated to show their relationship from different perspectives. Simple calculations may be performed using a light pen or cursor to indicate the region of interest. For example, cross sectional areas, section perimeters, or distances between selected points on the casting may be requested with the results written on the screen. Or circles may be inscribed within cross sections with the radii of the inscribed circles written on the screen. The user may edit sections by using the light pen or cursor to request that lines be moved or drawn in at new positions, to add padding, machine stock, core prints, risers, etc. The edited geometric model may be stored in a file for later use or a plot of the revised sections may be requested.

**GMSH2D** - Sections may be selected and edited to form a file of the geometry of interest for 2-D heat flow analysis of the freezing pattern. The model is built up from quadrilaterals. The selected section may be semiautomatically discretized into a mesh suitable for finite element or finite difference heat flow analysis.

**CSSP2D, SOLID**-Provisions are made for analyzing the freezing patterns of casting sections. Both finite element method (FEM) and finite difference method (FDM) programs are available. Program inputs are mesh files that divide a section into a grid of nodes and elements and material property files for the thermal properties of the alloy and mold. Properties of steel and sand are automatically used by the program; however, the user may override the program and enter data on another alloy/mold combination. Program outputs are files of computed temperatures at each node for several timesteps from the filling of the mold cavity. The output may be viewed directly in tabular form by reviewing the temperature files; however, the results are generally reviewed graphically using the simulation review routines in SIMREV.

**BCAST** - Two major functions may be performed within BCAST: development of a solid (3-D) model of the mold configuration (with ZEDIT) and automatic generation of a mesh of nodes and elements suitable for a 3-D heat flow analysis (with MGEN). The solid model is developed by describing the casting and the mold in terms of simple solid shapes (including bricks, cylinders, cones, hollow cylinders, and hollow cones). The 3-D model may be displayed on the screen in perspective or in top, side, or end views. The model may be automatically discretized into a finite element or finite difference mesh using a logic that is designed to fit the thermal

behavior typical of alloy castings in sand molds. Either brick (hexahedral) or wedge elements are formed. The mesh may be inspected layer by layer. The nodes and elements are listed in a file suitable for input to FEM and FDM programs (CSSP1 and SOLID).

GMSH3D, MG3D - Two programs are available for forming finite element and finite difference meshes under the control of the user. Only brick elements may be formed by these programs; however, the element sizes may be varied by the user. The nodes and elements are listed in files suitable for input to FEM and FDM analysis programs.

CSSP3D, SOLID - Both finite element and finite difference analysis routines are available for simulating in three dimensions the freezing patterns of alloy castings in sand molds. The programs are designed to work efficiently for the analysis of heat transfer involving solidification. Complex geometries may be considered. Program inputs are 3-D mesh files and property files. The mesh files give coordinates of the nodes, the arrangement of nodes into elements, and the type of material comprising the elements. The material properties for steel and sand are incorporated by the program unless the user overrides and enters data for another mold/alloy combination. The temperatures computed at each node of the model for each timestep from the filling of the mold are stored on a file tape. The data will be reviewed graphically using SIMREV.

SIMREV - Several routines are available for the graphical review of data files computed by the heat transfer simulation programs. These routines may be used to plot cooling curves at given locations, temperature gradient at fixed times (or at the stage of freezing), and freezing contours, i.e., isotherms at critical temperatures. These routines ease applying thermal criteria for predicting casting soundness that are compatible with computer analysis.

FITLGS - A gating system is analysed by simple routines to predict the velocity of metal in the runners and gates, the time to fill the casting completely or to an intermediate level, and the loss of temperature of the metal as it flows through the gating system. The user interactively describes the features of the gating system as input to the program. The results of the computations may be obtained in tabular or graphical form.

## 6. EXPERIMENTAL PROGRAM

In addition to developing the software summarized in the previous sections, experimental programs were designed and implemented to:

- Develop a data base of thermal properties for steel and molding sands to use as reliable input to the programs;
- Obtain thermal data on freezing patterns that could be used to check the validity of the programs;
- Develop casting soundness criteria that can be implemented with computer analyses.

Additionally, some fluid flow modeling was initiated to support the advanced fluid flow analyses under development. Preliminary measurements were made on the thermal properties of investment casting shell materials.

### 6.1 STEPPED PLATE CASTING EXPERIMENTS AND SIMULATION

The solidification simulations used in UPCAST are based both on the finite difference and the finite element techniques. Simulations are designed to perform either two-dimensional or three-dimensional analyses of the heat transfer during freezing and subsequent cooling of castings of complex shape. Axisymmetric castings are a special case handled by the two-dimensional analyses.

The quality of the data input to the programs for material properties and operating conditions will determine the quality of the results. However, the results of the analyses may be interpreted on different levels. The input data must be sufficiently accurate for the job at hand. A desire for more accurate data than necessary may remove the programs from being practical. Not only will the data be hard to collect, but complex input data will require more computer operations for their incorporation into the programs. Judgement is required to determine the level of accuracy to extend to the writing of the programs and the collection of data.

A stepped plate casting was the the major vehicle for obtaining the thermal and soundness data required. Two views of one of the stepped plate casting patterns are sketched in Figures 6-1 and 6-2. The pattern was made with several loose pieces so that several casting configurations could be made with one pattern. The dimensions of the plate castings are listed in Table 6-1. Castings were poured in no-bake sand and in green sand. All configurations were poured in no-bake sand.

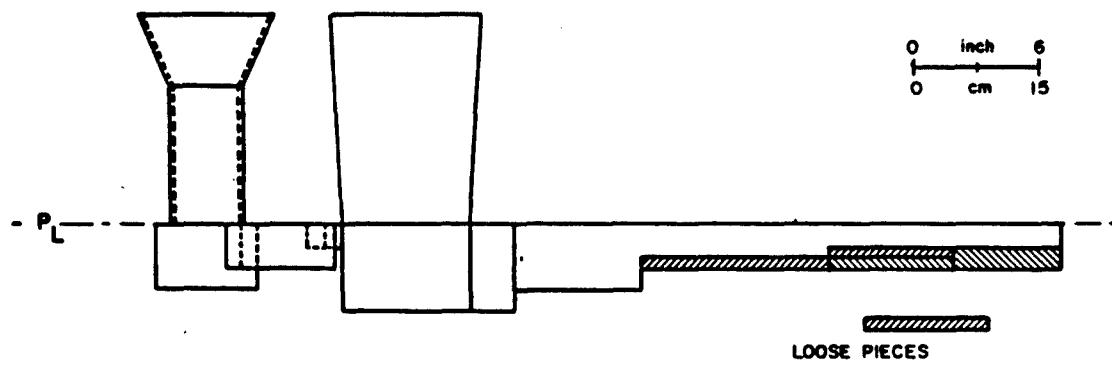


Figure 6-1: Side view of Blaw-Knox pattern for test plates.

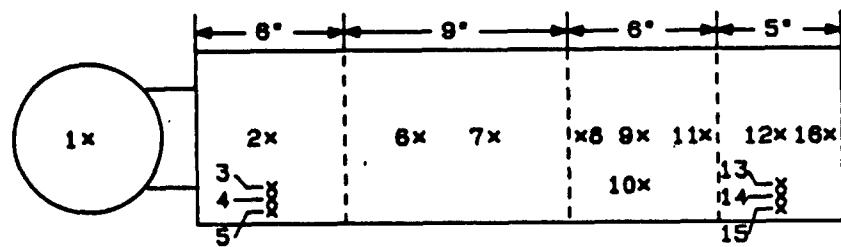


Figure 6-2: Positions of 16 thermocouples in plate castings.

Table 6-1: Dimensions of Test Plate Castings A - H.

CASTING	SECTION 1	SECTION 2	SECTION 3	SECTION 4	CHILL
DESIGNATION	6-in. lg.	9-in. lg.	6-in. lg.	5-in. lg.	
A	3	2	1.5	1	no
B	3	2	1	1	no
C	3	2	2	1	no
D	3	2	2	2	no
E	3	1.5	1.5	1	no
F	3	1.5	1	1	no
G	3	2	2	2	yes
H	3	1.5	1	1	yes

note: All castings were 7-inches wide.

Castings A-H were cast in no-bake sand.

Casting A, D, & G were cast in green sand.

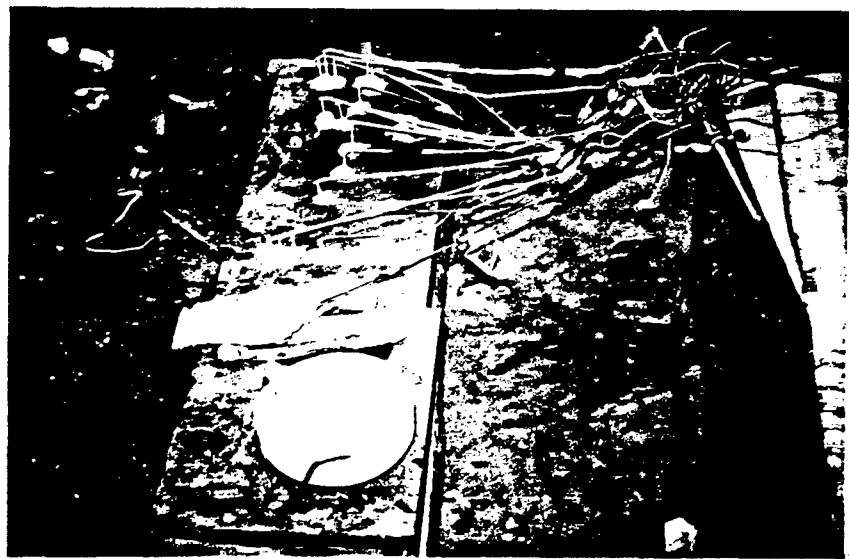
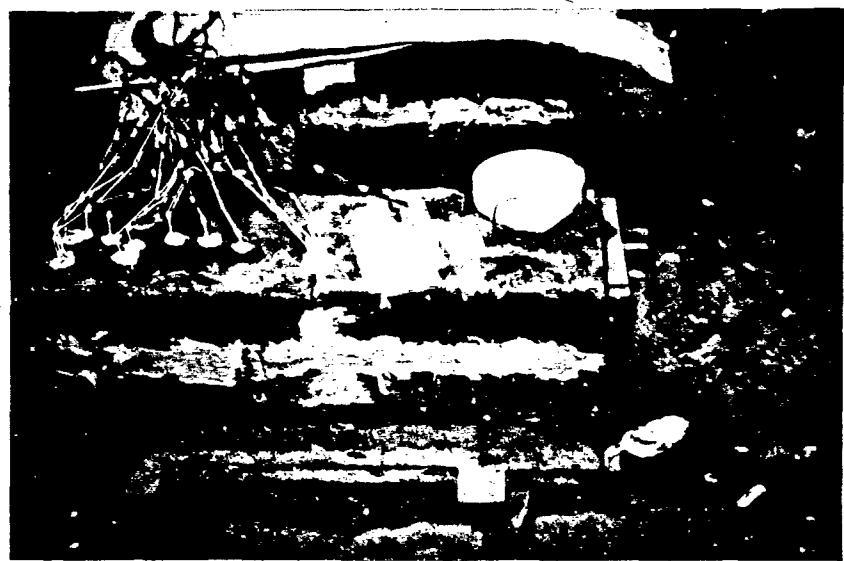
Only configurations A, D, and G were poured in green sand. The chemical compositions of the castings poured in no-bake sand are listed in Table 6-2. All casting designs were instrumented with sixteen to eighteen Pt/Pt-10%Rh thermocouples in fused silica protection tubes. The sixteen thermocouple positions are shown in Figure 6-2. The results were collected on strip chart recorders and entered into computer files for analysis using the digitizing table to transfer the thermocouple data. The plate castings were poured at Blaw-Knox Foundry in East Chicago, Indiana; at Lebanon Steel Foundry in Lebanon, Pennsylvania; and at the University of Pittsburgh. At Blaw-Knox, the steel was teemed from 28,000-pound bottom pouring ladles into no-bake sand molds. The experimental arrangement is illustrated in Figure 6-3. At Lebanon Steel, the steel was lip poured from 500-pound ladles into green sand molds. At the University of Pittsburgh, the steel was poured directly from a 300-pound tilt pour induction furnace into green sand and no-bake sand molds.

**Table 6-2:** Chemical Analysis of Stepped Plate Castings.

<u>ELEMENT</u>	<u>Test Plate Casting Designation</u>							
	<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>	<u>F</u>	<u>G</u>	<u>H</u>
C	0.25	0.27	0.30	0.25	0.26	0.28	0.30	0.28
Mn	1.00	1.20	1.26	1.18	1.28	1.25	1.08	1.20
Si	0.42	0.42	0.40	0.46	0.40	0.34	0.42	0.45
S	0.026	0.026	0.027	0.028	0.026	0.028	0.026	0.027
P	0.027	0.025	0.029	0.027	0.025	0.027	0.028	0.025
Cr	0.98	1.05	1.16	1.04	0.99	1.12	1.01	1.15
Ni	1.01	1.10	1.16	1.19	1.02	1.10	1.10	1.09
Mo	0.48	0.46	0.50	0.48	0.45	0.49	0.51	0.46

One concern of the project was determining the thermal parameters for the different types of molding sand. From analysis of the results of temperature measurements taken on the same casting geometries and the same steel composition poured into different mold materials it was concluded that *changes in cooling behavior as a result of changing from no-bake to green sand were smaller than the changes that occur due to normal variability in commercial foundry practice*. In a practical sense, the difference in thermal behavior of steel castings poured in green sand and no-bake sand is not significant and need not be included in computer models directed at predicting casting soundness.

The programs were provided with reasonable thermal data (selected from property measurements reported in the literature) and then the programs were used to predict the cooling behavior of the experimental castings. The results of the analyses were compared with measured cooling curves and temperature gradients. The thermal properties used in the computations were continually modified until a set of thermal properties which was reasonable and gave good agreement with the



**Figure 6-3:** Instrumented plate castings at Blaw-Knox foundry.

measured behavior was determined. Then, the same properties were used to predict the cooling behavior of another plate casting configuration. Again, the results of computations were compared to the experimentally determined cooling curves and temperature gradients. When the agreement obtained indicated a set of data had been selected that would yield consistently good predictions of casting cooling pattern and cooling rates, the data was accepted and incorporated into the materials property data file.

Based on the experimental results on the plate castings and the comparison with computer simulation, the thermal data in Table 6-3 were determined to adequately represent the freezing behavior of steel cast in sand molds in relation to computer simulation of foundry practice. In addition to the data from plate castings the thermal data is based on experiments conducted to determine the distribution of the heat of fusion over the freezing range of low alloy steel.

**Table 6-3: Thermal Properties of Steel & Sand for Simulation.**

Steel			
Temperature deg. F	Thermal Conductivity BTU/ft-hr-deg. F	Density BTU/ft <sup>3</sup>	Heat Capacity BTU/pound-deg F
2705	19	489	0.19
2669	18	489	3.10
2605	17	489	0.38
50	15	489	0.15
Sand			
	0.45	95	0.25

The second type of experiment, as reported in Chapter 7 of Volume II, is the measurement of the cooling curve of the alloy in an insulating mold. Then, the temperature is plotted as a function of the square root of time, and the ratio of the cumulative thermal arrest at a given temperature in the freezing range to the

cumulative thermal arrest at the solidus temperature is plotted as a function of temperature. For low alloy steel, this technique approximates very well the distribution of the release of latent heat through the freezing range. The results for armor steel, as for other alloys measured, indicate that most of the latent heat is liberated near the liquidus temperature. Reference to the data presented in Table 6-3 indicates that adequate prediction of freezing and cooling is achieved by using data that puts 82 percent of the latent heat in the first 36 percent of the freezing range.

An example of the cooling curves measured in plate casting A are shown in Figure 6-4. The solid lines are measured cooling curves at positions along the centerline (1, 2, 6, 7, 12, and 16 in Figure 6-2). The dotted lines are cooling curves computed for equivalent positions. Note that freezing in this plate is progressive from plate end to riser. Also plotted in Figure 6-4 are the results of finite element simulation of the freezing pattern in plate A. The agreement is very good. Agreement with other configurations is also good. These results were obtained with the three-dimensional solidification simulation routines. Good agreement could not be obtained using the two dimensional analysis programs. Good correlation could be obtained with the blocky sections or with the thinner plates, but not with both, simultaneously.

The analysis of the freezing pattern in plate casting A, which had steps of 3, 2, 1 1/2, and 1 inch fed by a 6 inch diameter riser, indicated the limitations of a 2-D analysis. The geometry of the plate castings offers symmetry around the central axial plane. Since the primary concern, from a feeding standpoint in armor steel, is centerline shrinkage, it would appear that a 2-D analysis of the central axial plane would give the information required. Indeed, a 2-D analysis is sufficient to indicate that the freezing pattern is progressive. However, a two dimensional analysis cannot predict the relative rate of cooling in the plate steps of different thicknesses and cannot adequately predict the temperature gradients. The difficulty stems from the changing width to thickness ratios of the riser and the steps (1, 2.3, 3.5, 4.7, 7). A casting engineer accustomed to comparing volume to surface ratios or cross-section to perimeter ratios (modulus) would obtain the same result. The importance of heat transfer in the width direction varies significantly along the axial direction of the casting. This factor cannot be neglected. A three-dimensional analysis correlates very well with experimental data and provides good data on relative cooling rates and thermal gradients along the axis of the plate casting A.

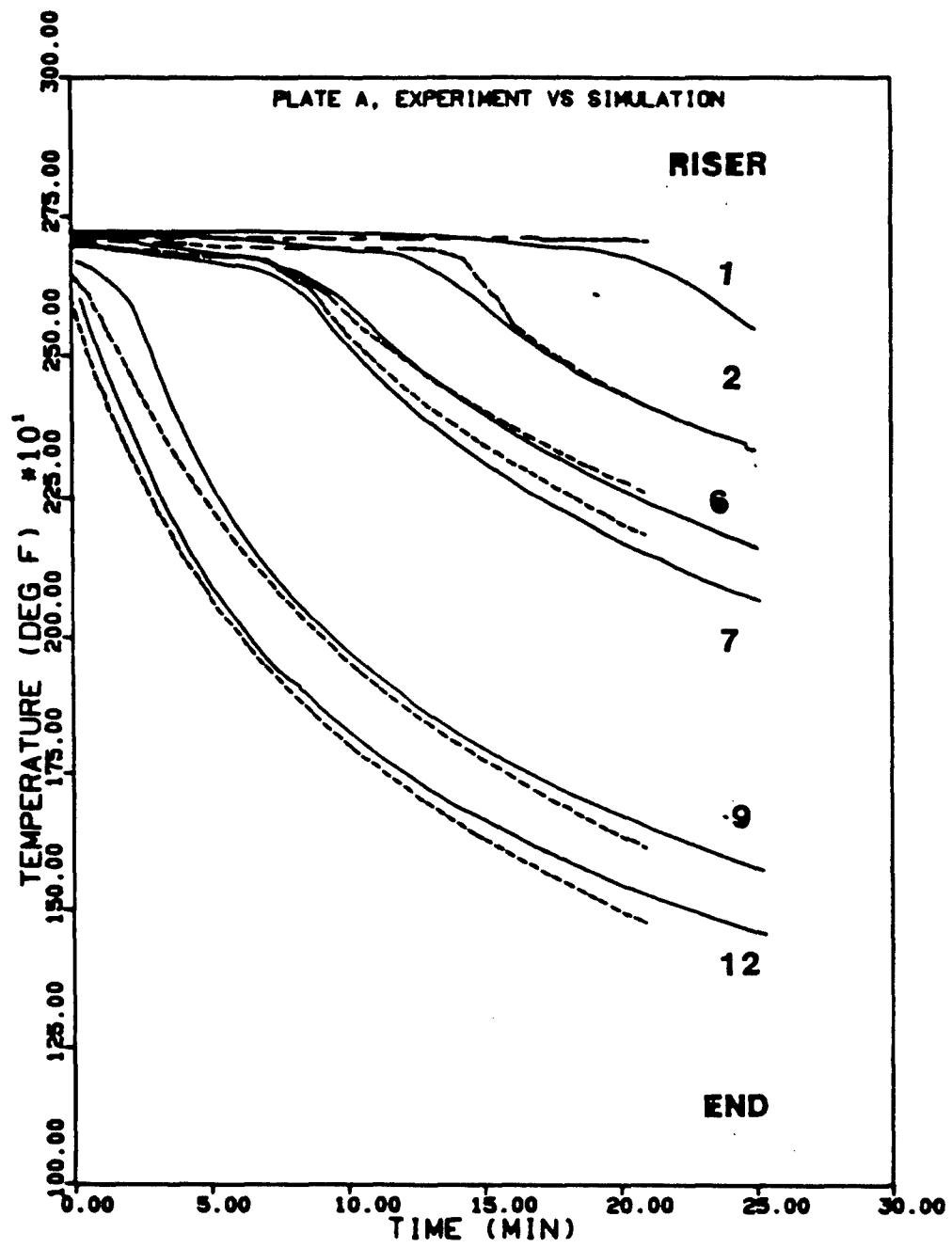


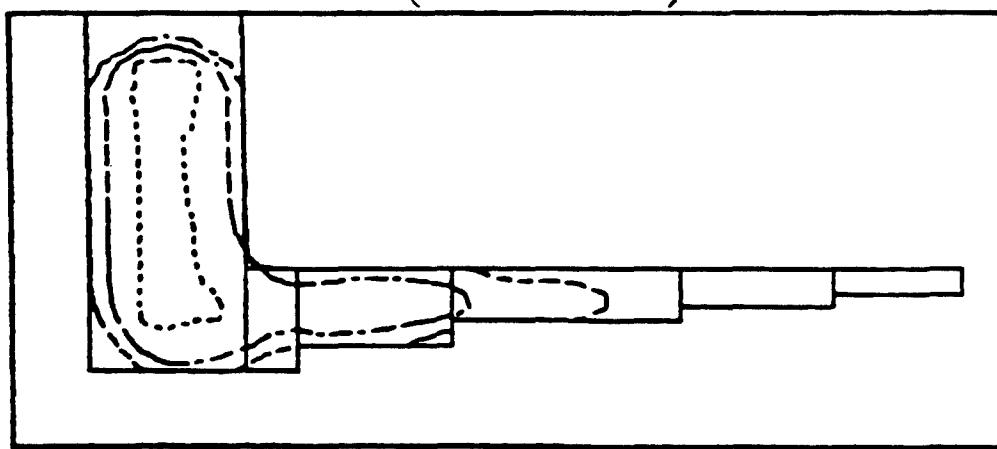
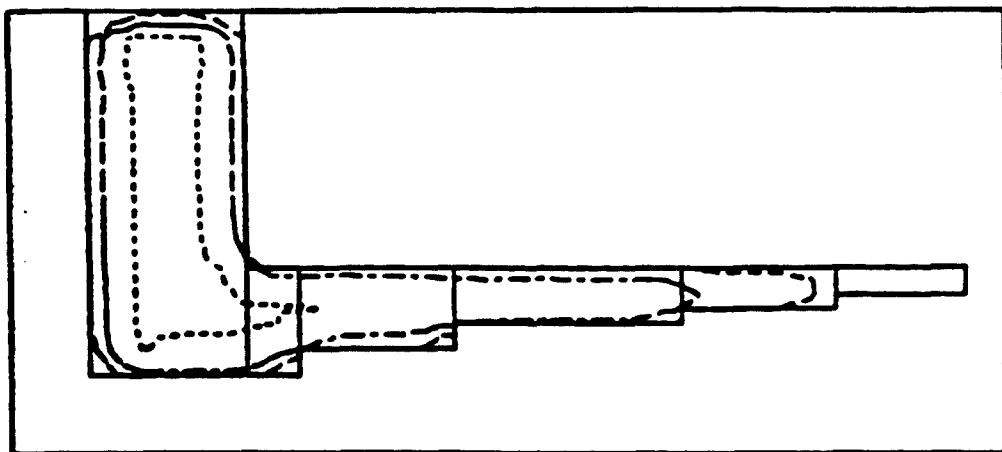
Figure 6-4: Computed (dash) and measured (solid) cooling curves, plate A.

## 6.2 CASTING SOUNDNESS CRITERIA

The most obvious and necessary criterion for casting soundness is that the casting freeze progressively. Regions away from a riser should finish freezing before regions adjacent to a riser; and the riser should finish freezing last. Solid bridges should not be allowed to separate still mushy regions of the casting from a supply of feed metal. The results of computer simulation of freezing patterns in castings can be interpreted, readily, in terms of progressive freezing. UPCAST eases review of the temperature data by following the progress of critical isotherms, by mapping regions of solid, partial solid, and liquid, or by plotting number codes to indicate relative temperature. A contour plot showing the liquidus, solidus, and an intermediate isotherm are plotted in Figure 6-5 at two times during the freezing of Plate A. Both Figure 6-4 and Figure 6-5 indicate freezing is progressive from the thinnest to the thickest plate. The computer results also show that the riser neck was inadequate and there would be bridging off of feeding for the three-inch plate. The computer simulations were developed to a point to be a good predictor of the progressiveness of freezing pattern. Based on the stepped plate casting experiments, the thermal data base for steel/sand mold casting is adequate and the model has reasonable accuracy.

Progressive freezing is a necessary but not a sufficient condition for producing sound, high integrity castings. There must be sufficient driving force to cause liquid to flow between dendrite arms in the partially solidified regions. It is necessary to achieve a sufficiently large temperature gradient during freezing to allow interdendritic feeding. What is important here is the temperature gradient in the feeding direction at the time the region of interest goes through the freezing range. For a particular alloy, section size, and desired degree of soundness, it was postulated that a critical temperature gradient would be required. Through the experimental work of this program, the concept of a critical temperature gradient for soundness was shown to hold for armor steel cast in sand molds. Figures 6-6 and 6-7 illustrate the experimental and computed temperature gradients in plate castings A and D at the time each location was undergoing freezing. Firstly, the agreement is good. Secondly, in plate A, which is a sound casting, the gradient never goes below 6 degrees Fahrenheit per inch. The gradient approaches zero in Plate D. Thermal gradient is a parameter that may be used as a soundness criterion with computer analyses. Both the progressiveness of freezing pattern and the level of temperature gradient in the freezing range are soundness criteria suitable for implementation with computer analysis of the solidification process in castings.

The correlation between experimental and computational phases of the program is



**Figure 6-5:** Isothermal contours predicted for plate casting A.

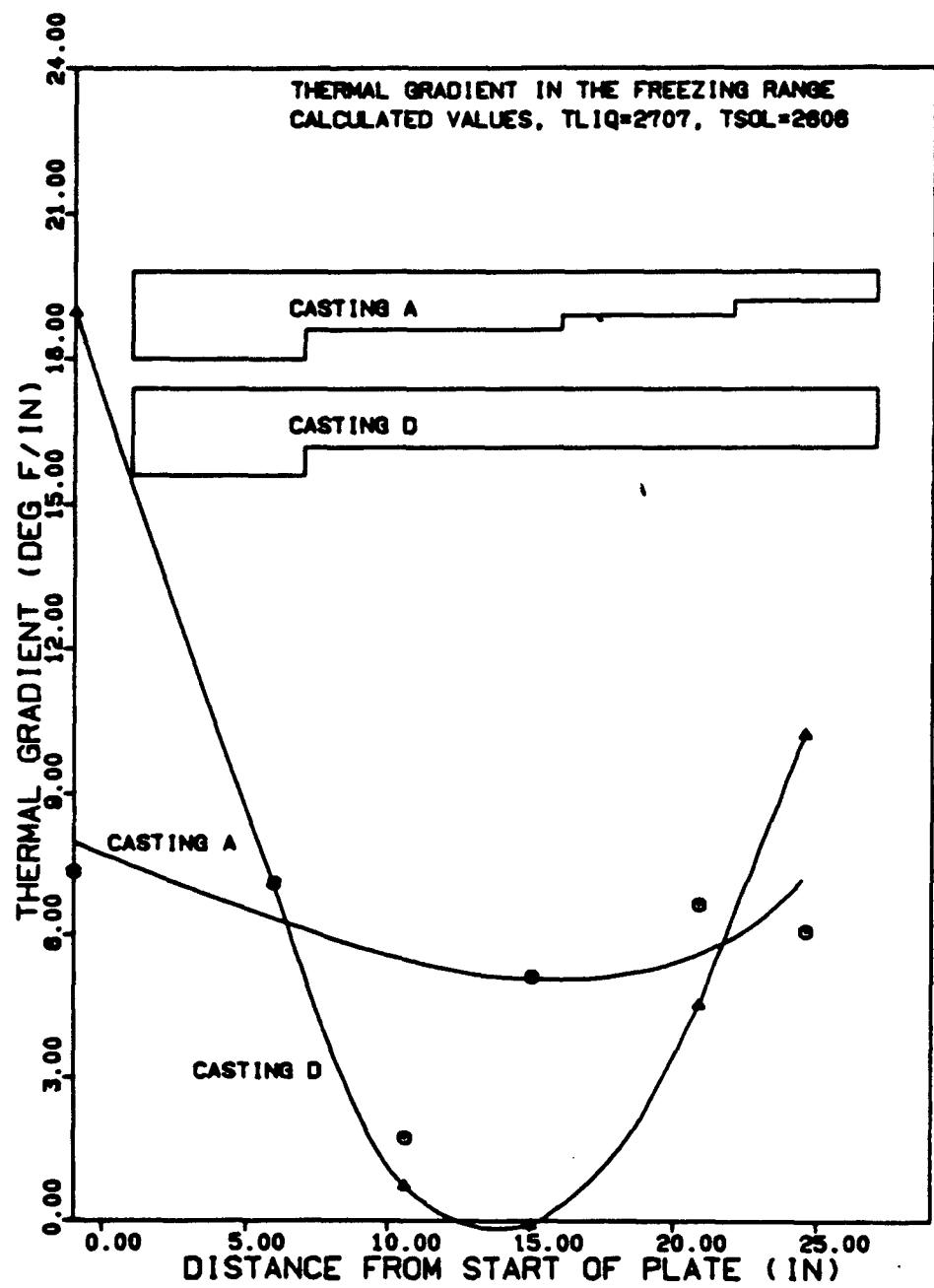


Figure 6-6: Temperature gradients in freezing range, computed.

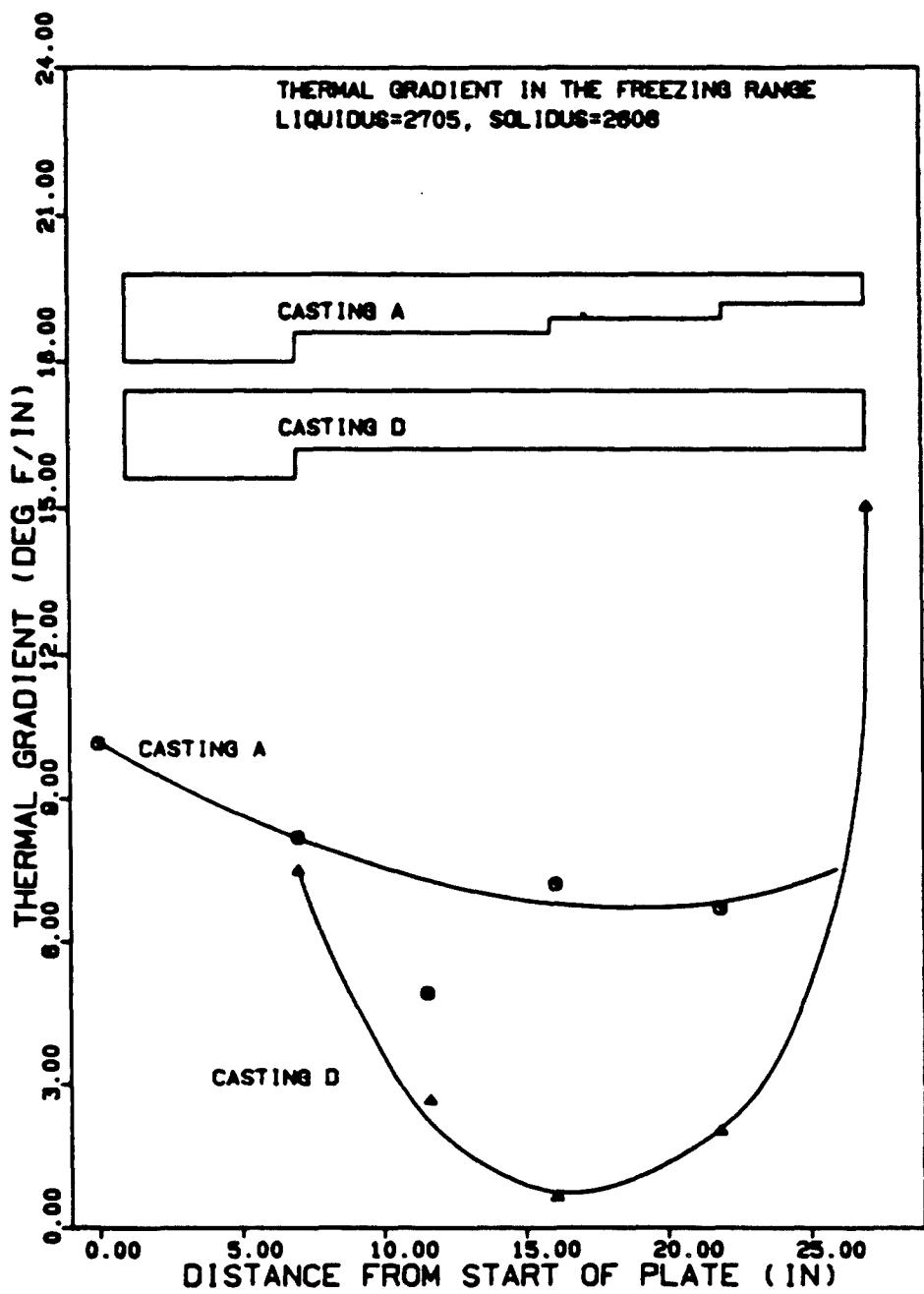


Figure 6-7: Temperature gradients in freezing range, measured.

described in a paper that was prepared for the Steel Founders Society of America and is included as Exhibit III. A paper that presents the mathematical basis of the analysis techniques in relation to casting problems was published in the Journal of Metals and is included as Exhibit IV. Another paper that discusses the development of soundness criteria and also leads into the subject of the next chapter, i.e. fluid flow analysis in mold cavities, was presented at a solidification science symposium at the Massachusetts Institute of Technology and is included as Exhibit V.

### **6.3 TORSION BAR HOUSING CASTINGS**

In addition to the stepped plate castings over 100 torsion bar housing castings for the Abrams (M1) tank were poured at the Blaw-Knox and Lebanon Steel foundries. Several of the castings were instrumented and some were radiographed. All were visually inspected before and after rough machining and after heat treatment. Altogether six different configurations for risers and chills were tested on the torsion bar housing castings. None of the risering configurations presented unacceptable shrinkage defects in the torsion bar housing castings. After completion of the experimental phase of this work, some castings were shown to have unacceptable defects. However, these were not shrinkage porosity defects.

Five different mold configurations were used to make the torsion bar housing castings at the Blaw-Knox steel foundry in East Chicago, Indiana. First, two demonstration castings were cast, tested, and shipped to TACOM for approval. Later, over one hundred torsion bar housing castings were poured, cleaned, visually inspected (as a minimum), and shipped to TACOM. Thermocouples were placed in six of the molds in 18 locations each to obtain detailed cooling rate data. Eight torsion bar castings were cast in green sand at Lebanon Steel Foundry. Two were instrumented for temperature measurement. The castings that were not instrumented were shipped to TACOM.

#### **6.3.1 No-Bake Sand**

At Blaw-Knox Foundry the torsion bar housing castings were cast four to the mold in a twelve foot diameter circular flask. The castings were organized into groups of twenty, i.e., five flasks were molded and poured for each group. In the first group all castings were of the same mold configuration (riser placement and gating) as employed in the two demonstration castings (configuration I). In the second through fifth groups, the mold configuration for two of the housing patterns on the match plates were kept the same as configuration I, and the mold configuration for the other two housing patterns was changed. Thus, a total of

sixty castings were made with configuration I and ten each were cast with configurations II - V.

Descriptions of the different mold configurations are given below.

**Configuration I:** The control configuration utilized four side risers, two top risers, and no chills. Figures 6-8, 6-9, and 6-10 illustrate the mold arrangement, pattern, and rough casting for configuration I. The central drag runner filled two ingates for each torsion bar housing casting. The gates were attached to the 4 1/2 inch and 6 1/4 inch side risers. The runner was fed through a central downspur. The large bore in the housing was cored through its entire length (note the core prints on the pattern). The small bore was cored only halfway. A 4 1/2 inch diameter riser was placed on top of the large plate. Three 3 1/2 inch diameter risers were used to feed the walls of the large cylinder. One was a top riser and two were side risers (note the cope view of the pattern). Figure 6-10 shows rough castings just as they were removed from the mold after pouring.

**Configuration II:** One 3 1/2 inch side riser on the large cylinder was removed and a steel chill was located at the former riser contact.

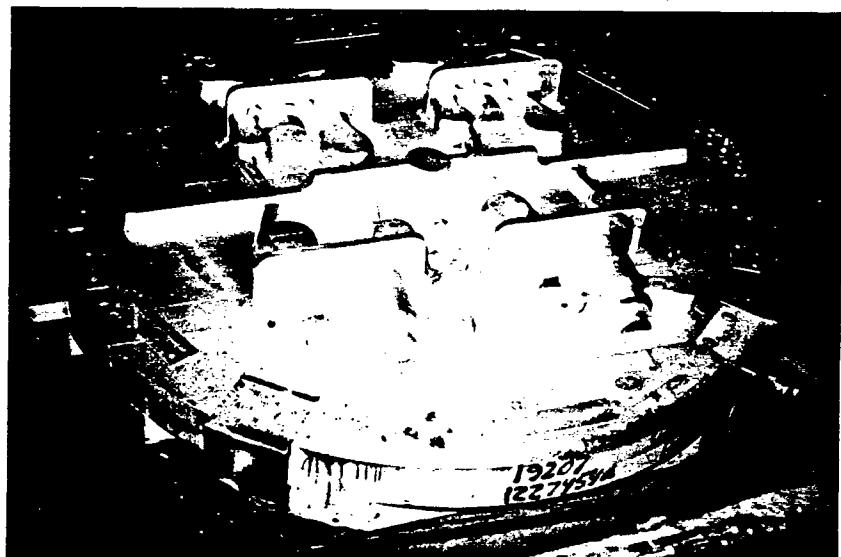
**Configuration III:** Both 3 1/2 inch side risers on the cylinder were removed and a chill was placed on the drag directly below the top riser.

**Configuration IV:** The 3 1/2 inch top riser was removed. The two 3 1/2 inch side risers on the cylinder were replaced, and a cope and drag chill were placed on the large cylinder between the former position of the large riser and the large plate.

**Configuration V:** The top riser of the large cylinder was replaced and the 4 1/2 inch diameter top riser on the plate was removed. A vent was located at the former riser contact.

### **6.3.2 Green Sand**

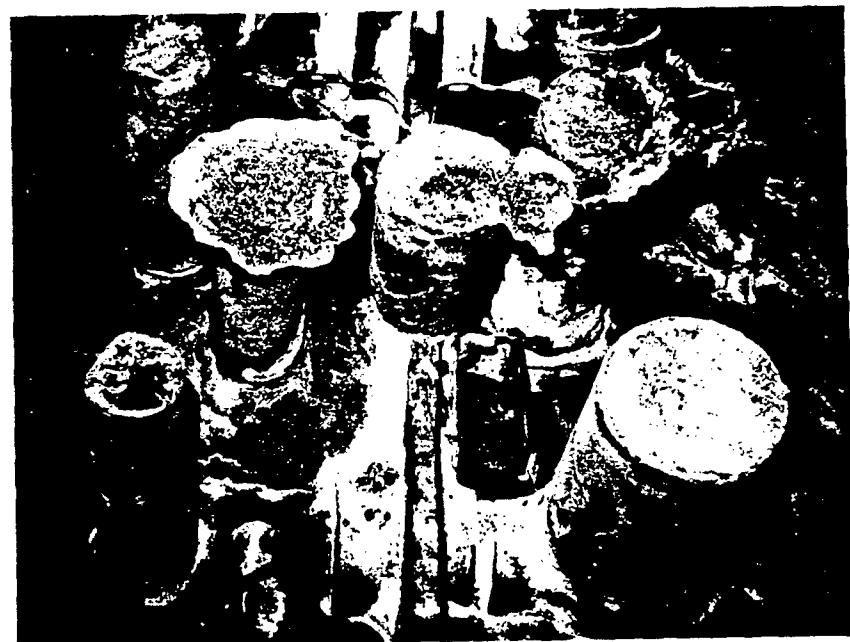
At Lebanon Steel Foundry the mold configuration used to cast eight torsion bar housings was similar to the control group at Blaw-Knox. The coring was similar. Four side risers were used and the casting was gated through the risers. One casting was poured in each mold. Two castings were instrumented for thermal analysis, each at eighteen locations.



**Figure 6-8:** Drag pattern & mold for configuration I.



**Figure 6-9:** Cope pattern (one set) & mold for configuration I.



**Figure 6-10:** Rough castings (configuration II) right after shakeout.

## 7. RESEARCH EXTENSIONS

### 7.1 FLUID FLOW IN MOLD CAVITIES

The application of the mechanical energy balance in forms such as Bernoulli's equation has been long practiced in casting analyses. The mechanical energy balance was the approach adopted in the FITLGS routines of UPCAST to compute the filling times of mold cavities. This approach is adequate for describing the overall behavior of gating systems. It will not provide information on the filling stages of either the gating system or the mold cavity nor will it provide details on the flow pattern in either the gating system or the mold cavity.

The Bernoulli approach was extended beyond the content of FITLGS to allow computer analysis of the distribution of the metal entering the mold cavity from a gating system with multiple ingates. This program allows computing of the percent of metal that flows into the mold cavity from each ingate. This has been a long standing concern in the foundry when more than one gate leads off of the same runner. It is common for much more metal to enter the mold cavity from one of the gates. In fact some gates may be bypassed completely. The computer program affords a tool which a foundryman may design a gating system to give uniform filling through multiple ingates.

The pattern of filling the mold cavity, including details on the localized velocities and pressures and the presence of vortices, waves, and dead zones, would be of broad interest to foundrymen/casting engineers. However, this open channel transient flow cannot be treated by Bernoulli's equation or other common methods of fluid dynamics that usually treat steady state flow in closed systems (such as pipes). Work on flow modeling in nuclear reactor applications, centered in the Department of Energy, has led to numerical techniques that may be adopted to analysis of materials processing. Under this project we have adapted some of those techniques to analysis of flow in castings. As example, steps in the filling of plate casting A as modeled with the computer analyses are presented in Figure 7-1.

The primary method adopted for analysis of mold cavity filling is the marker and cell (MAC) technique. The marker and cell technique uses the finite difference method to solve the Navier-Stokes equation. As in other numerical techniques of simulation, the casting cavity is first discretized, i.e., divided into elements or cells, and nodes are associated with each cell. Imaginary markers are considered by the program to enter with the fluid. The markers are treated as if they are the same density as the molten alloy and they are assumed to move with the same velocity in

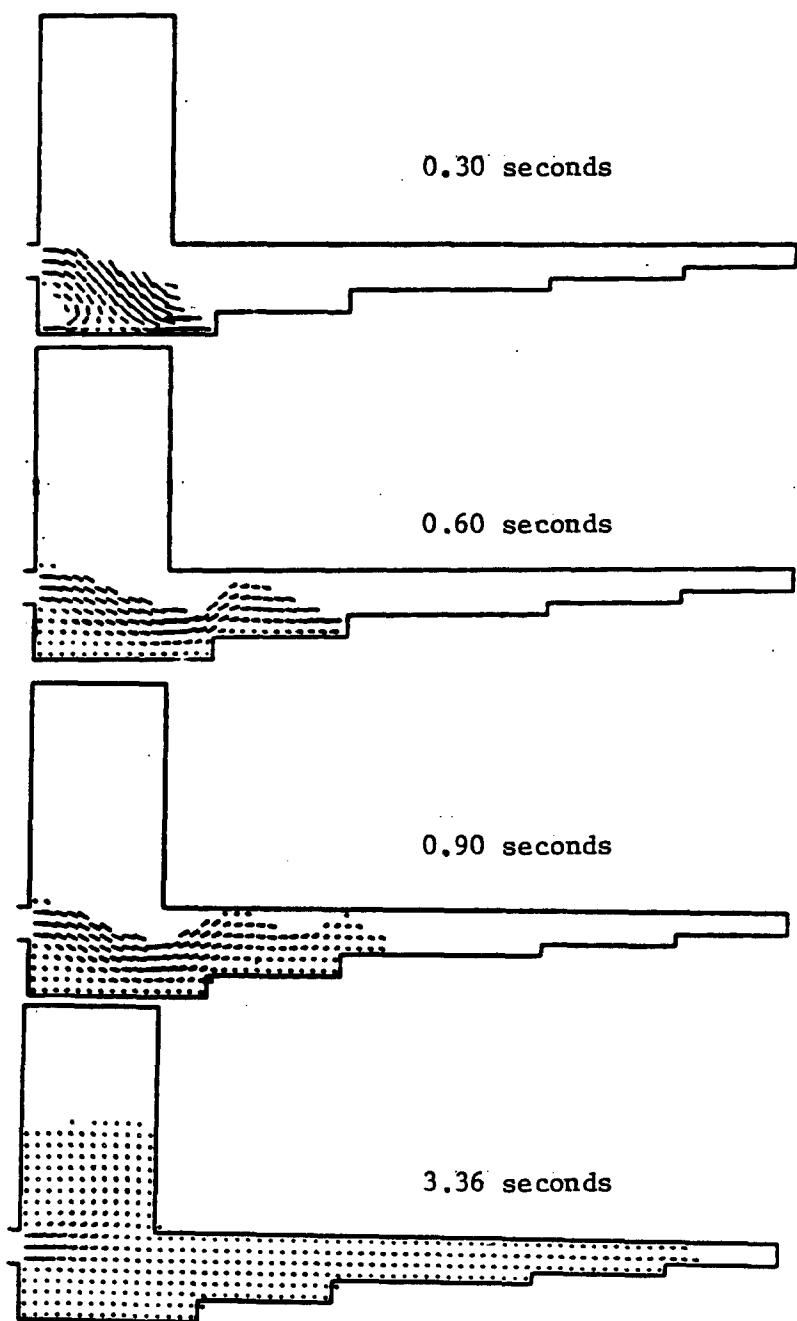


Figure 7-1: Computed filling pattern for plate casting A.

magnitude and direction as the fluid. As the markers enter a cell, it is considered to fill with fluid. Filled cells that are bounded by empty cells are considered to be on the fluid boundary. The program makes calculations on the full cells and applies the boundary conditions to cells on the fluid boundary. The procedure predicts the pattern of filling, local velocities and pressures, places where vortices form, dead zones, etc.

The results are surprisingly accurate. In this project the results of computer computation were checked by comparison to water models. Plexiglass models of gating systems and mold cavities were made and water was "poured" into the gating system. High speed movies were taken of the filling of the transparent molds and the results were compared with the predictions of the marker and cell technique computations. Of particular interest was the physical and computer modeling of the filling pattern of plate casting A. The water model and the marker and cell computations agreed closely in flow pattern, including order of filling, formation of waves, formation and movement of vortices, and formation and decay of surface waves. The modeling provided valuable insight into the filling pattern of a complex mold cavity. Two papers describing the advanced fluid flow modeling techniques have been included as Exhibits VI and VII.

## 7.2 INVESTMENT CASTING

Conceptual and experimental work to apply computer design and analysis techniques to investment casting was initiated. One of the assumptions made in the heat flow simulation packages in UPCAST is that the metal enters the mold rapidly and then begins to cool and freeze. In some casting geometries and section sizes, this would not be a good assumption. As the metal enters the mold, it loses heat to the mold material and the mold material is preheated in the region of the ingate. The proper analysis of this type of casting depends upon the combined application of the fluid flow techniques being developed under this program and the heat flow programs in UPCAST. A coupled heat and fluid flow simulation is considered important to the analysis of investment casting. In investment castings section sizes are generally very thin in comparison to their width and length and the simultaneous flow and cooling of the molten alloy is critical to the processing and to the control of the metallurgical structure. Further considerations in investment casting including discussion of investment shell material thermal properties are given in Exhibit V.

# I. SIMULATIONS IN THE DESIGN OF SAND CASTINGS

Robert A. Stoehr  
848 Benedum Hall  
University of Pittsburgh  
Pittsburgh, Pennsylvania 15261

## ABSTRACT

Programs using the finite difference and finite element methods are available to aid the casting engineer in the design of sand mold castings, but further development will undoubtedly extend their usefulness. The desirable attributes of such a program, including simplicity, realism, flexibility, accuracy, stability, and ease of interpretation, are often attainable only at the expense of each other or at the expense of the economic justifiability of the simulation. Some compromises to reduce these conflicts are suggested and some instances where currently available programs are being used in the design of production castings are reported.

\* Originally presented and published: R. A. Stoehr, "Simulations in the Design of Sand Castings," Modeling of Casting and Welding Processes, eds., H. D. Brody and D. Apelian, T.M.S.-A.I.M.E., Warrendale, PA., 1981.

## I.1 INTRODUCTION

Computer modeling of sand castings can make an important contribution to the economical production of new designs. The objective is to assist the casting engineer in translating the drawings and specifications for a finished part into the proper design of the mold and the proper casting practice, including, as shown in Figure I-1 (The typical sand casting to be modeled is a complex assemblage of numerous materials, which dictates to a large degree the complexity of the mesh used to represent it):

1. Placement of gates and risers.
2. Selection of sands and other mold materials.
3. Selection and locations of chills.
4. Allowances for shrinkage, machining tolerances, and drafts.
5. Choice of pouring temperatures, hot-topping practices, etc.

The program should enable the casting engineer to determine the location of liquidus and solidus, the temperature at any point within the casting, the local cooling rate, the local temperature gradient, and the rate of liquidus and solidus advance, all at appropriate time intervals within the casting process.

With this knowledge, the casting engineer can draw conclusions concerning the likelihood of the formation of voids and porosity, cracks, chill zones, microsegregation, and certain microstructures.

## I.2 BACKGROUND

Development of digital computer simulation of the casting process has been going on for some years and application of various forms of the finite difference method to solidification has been described by a number of authors.<sup>1-13</sup> In early work, Schniewind<sup>1</sup> developed a method for calculating the movement of the solidification front between nodal points in a pure metal.

Of particular note is the pioneering work of Henzel and Keverian<sup>2</sup> in using a large finite difference mesh (200 to 1000 nodes) to represent the irregular shapes found in real sand castings. They adapted a general purpose transient heat transfer program which solved the implicit form of the finite difference equations by the Gauss-Seidel technique, and they reported good agreement between their calculated and experimentally measured temperatures and cooling times.

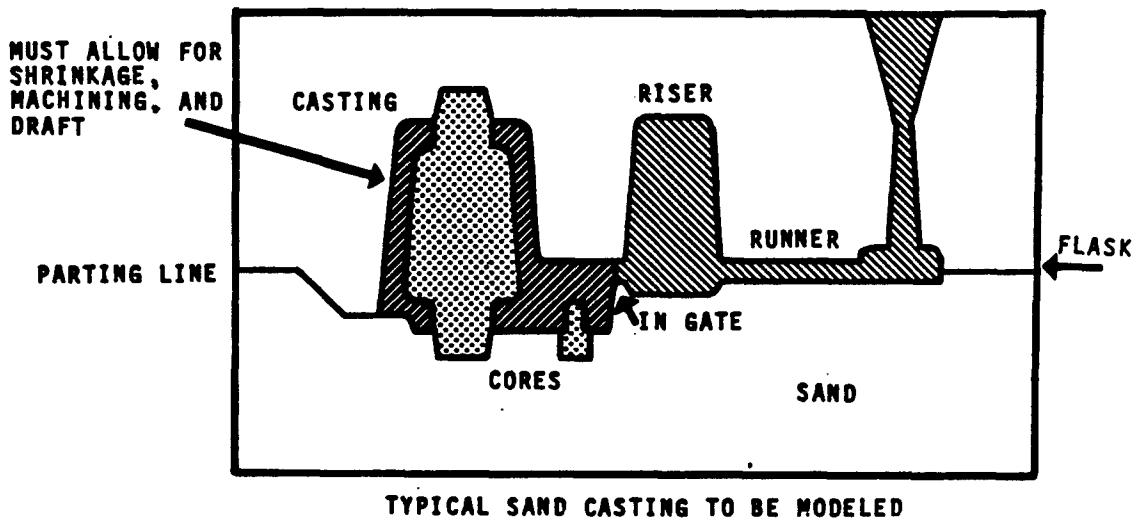


Figure I-1: Typical sand casting is a complex assemblage.

Mizikar<sup>3</sup> modeled heat flow in a continuously cast steel slab using a one-dimensional finite difference technique. Lait, Brimacombe, and Weinberg<sup>4</sup> developed a one-dimensional finite difference heat transfer model for continuously cast steel which considered convective mixing and latent heat released throughout the solidification range.

Sciama<sup>5-7</sup> calculated temperature distributions in the solidification of various shapes such as cylindrical bars, plates, spheres, "L"s, "T"s and crosses. Narrone, Wilkes, and Pehlke<sup>8</sup> modeled the solidification of "L"-shape and "T"-shape low carbon steel casting utilizing an artificially raised heat capacitance to account for heat of fusion. Pehlke, Kirt, Narrone, and Cook<sup>9</sup> simulated the solidification of silicon brass and aluminum castings with a two-dimensional explicit finite difference method. Kirt and Pehlke<sup>10</sup> used the correlation of computer simulations with experimental results to determine thermal properties of materials and heat transfer coefficients. Jevarajan and Pehlke<sup>11</sup> modeled a casting with a chill.

Brody and Stoehr<sup>12-13</sup> have reported on a system for calculating solidification of very large alloy steel roll casting using a two-dimensional, axisymmetric finite difference program. This program accommodates up to 20 materials with temperature dependant properties, variable heat transfer coefficients that adjust for the formation of an increasing gap between the metal and mold as solidification progresses, and the ability to simulate the "double pouring" of rolls. This program also features simplified entry of complex two-dimensional meshes, built-in diagnostics that check the input data for internal consistency, and simplified methods for producing pictorial output on a line printer.

Although most of the work in the casting industry has been done by the finite difference method, some people have been looking at the finite element method. Das<sup>14</sup> has advocated the flexibility and versatility of the finite element method for application to the design of castings. Orivuori<sup>15</sup> and Brunch and Zyvoloski<sup>16</sup> reported the use of the finite element method to simulate heat conduction in bars and other shapes. Comini, Del Guidice, Lewis and Zienkiewicz<sup>17</sup> examined nonlinear heat conduction by the finite element method with special reference to a change of phase. They tested their work on the solidification of an infinite slab and a corner region. Morgan, Lewis, Zienkiewicz<sup>18</sup> developed a more numerically stable finite element solution. They incorporated the heat of fusion into the enthalpy, representing it by a smooth curve. The accuracy of using finite element solutions to the transient heat conduction equation has been verified by Donea<sup>19</sup> and Cella<sup>20</sup>. Mathew and Brody<sup>21-23</sup> have used the finite element method to model heat flow and thermal stresses in the solidification of axisymmetric continuous casting.

### I.3 CURRENT STATUS AND THE NEED FOR FUTURE DEVELOPMENT

As a result of this work, programs are now available, both commercially and from private sources, which can be used for the simulation of the casting process in sand molds. No attempt will be made to catalog them here because this will be done in other papers at this conference. Nevertheless, their use is not widespread in the foundry industry.

At the University of Pittsburgh we have developed several two- and three-dimensional programs for the simulation of sand mold castings using either the finite element<sup>24,13</sup> or the implicit finite difference method<sup>12,13</sup>. Some designers are using these programs at the present time to test proposed sand mold designs. In order to make the programs more widely useful, we are continuing to develop them with the user's needs in mind.

In this paper we will attempt to outline the things which must be done to make computer simulation a universally accepted tool in sand mold casting design.

In developing a program for modeling the solidification of sand mold castings, one tries to attain certain desirable attributes, commonly including:

1. Simplicity from the user's viewpoint.
2. Realistic representation of the casting and mold design.
3. Flexibility to represent a wide variety of designs.
4. Accuracy of results as indicated by:
  - a. Agreement with analytical solutions.
  - b. Agreement with experimental results.
5. Stability.
6. Easily interpreted results.
7. Economically justifiable costs.

Unfortunately, attaining any one of these attributes is often in conflict with attaining the others.

### I.3.1 Simplicity vs. Realistic Representation

In both the finite difference method (FDM) and the finite element method (FEM), geometrically complex castings are represented by dividing the space which contains the mold and casting into a number of subdivisions called elements. The elements themselves must be simple geometric shapes which completely fill the space. In two-dimensions they are usually triangles and/or quadrilaterals. In three-dimensions they are usually tetrahedrons, wedges, and/or hexahedrons. These elements are generally bounded by straight lines and planes. (As illustrated in Fig. I-2 the casting and rigging must be divided into a mesh consisting of tetrahedral, wedge, or hexahedral-shaped elements when using the finite difference or finite element methods. The mesh must follow the contours of the mold and the casting in such a way that no element contains more than one material.)

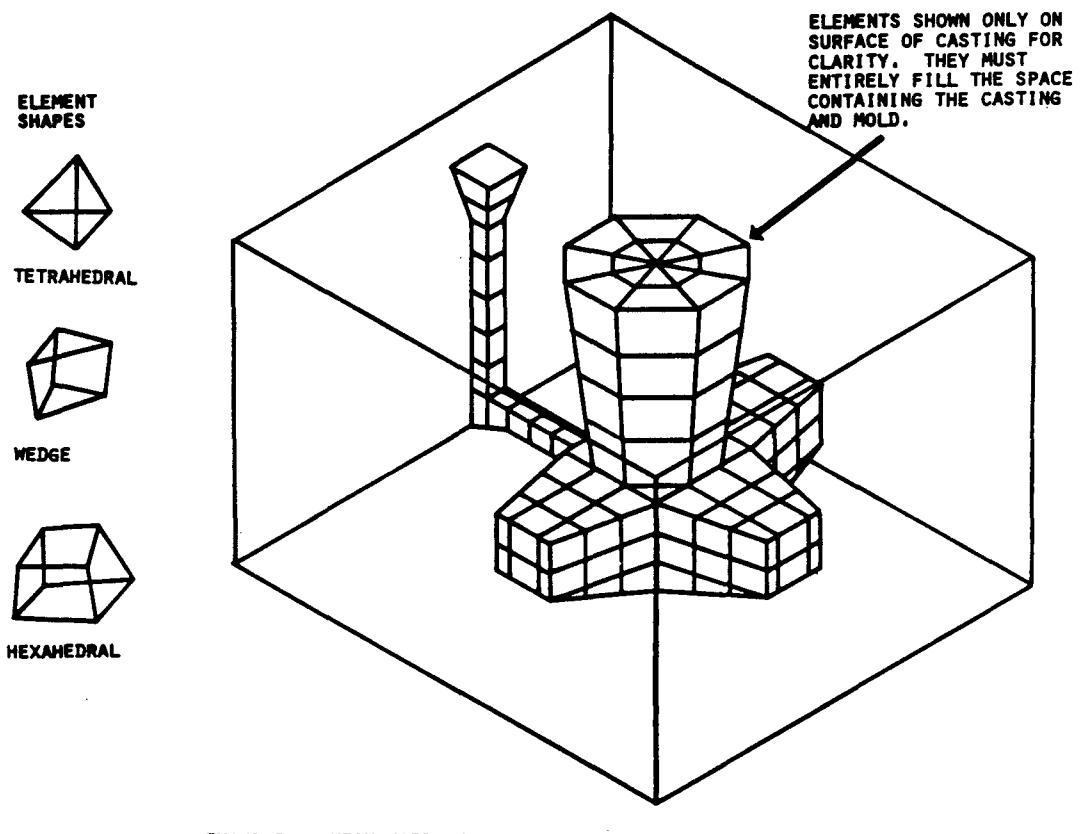
The number, exact shape, and location of these elements are generally chosen with two ideas in mind:

1. The boundaries between element should follow the actual boundaries of the materials as closely as possible (dictated by the requirement that each element should contain only one material).
2. Their spacing determines the spatial resolution of the temperatures to be calculated.

Geometrically speaking, the major difference between the finite difference method is that in the FEM the temperatures are calculated at nodes which are located at the corners of the elements, while in the FDM the temperatures are calculated at nodes which are located at the centers of the elements.

Generating the mesh of elements and nodes is not a trivial chore. Typically, representation of a casting and mold design in two-dimensions will require 30 to 50 rows of elements in each direction for a total of 900 to 2500. Including the third dimension with the same resolution increases the number to 27,000 (for a 30 x 30 x 30 mesh) or to 125,000 (for a 50 x 50 x 50 mesh). Even the 27,000 elements stretch the the capability of most computers.

One may ask whether such resolution is necessary. Our experience indicates it is. For example, on large steel cast rolls, a difference of 1-in. in the neck radius in a mold with an overall mold radius of 40-in. makes the difference between a sound casting and a casting with voids in a critical location (Fig. I-3). The voids form, of course, when metal in the cope neck freezes before solidification in the body is complete.



**Figure 1-2:** The casting and rigging must be divided into a mesh.

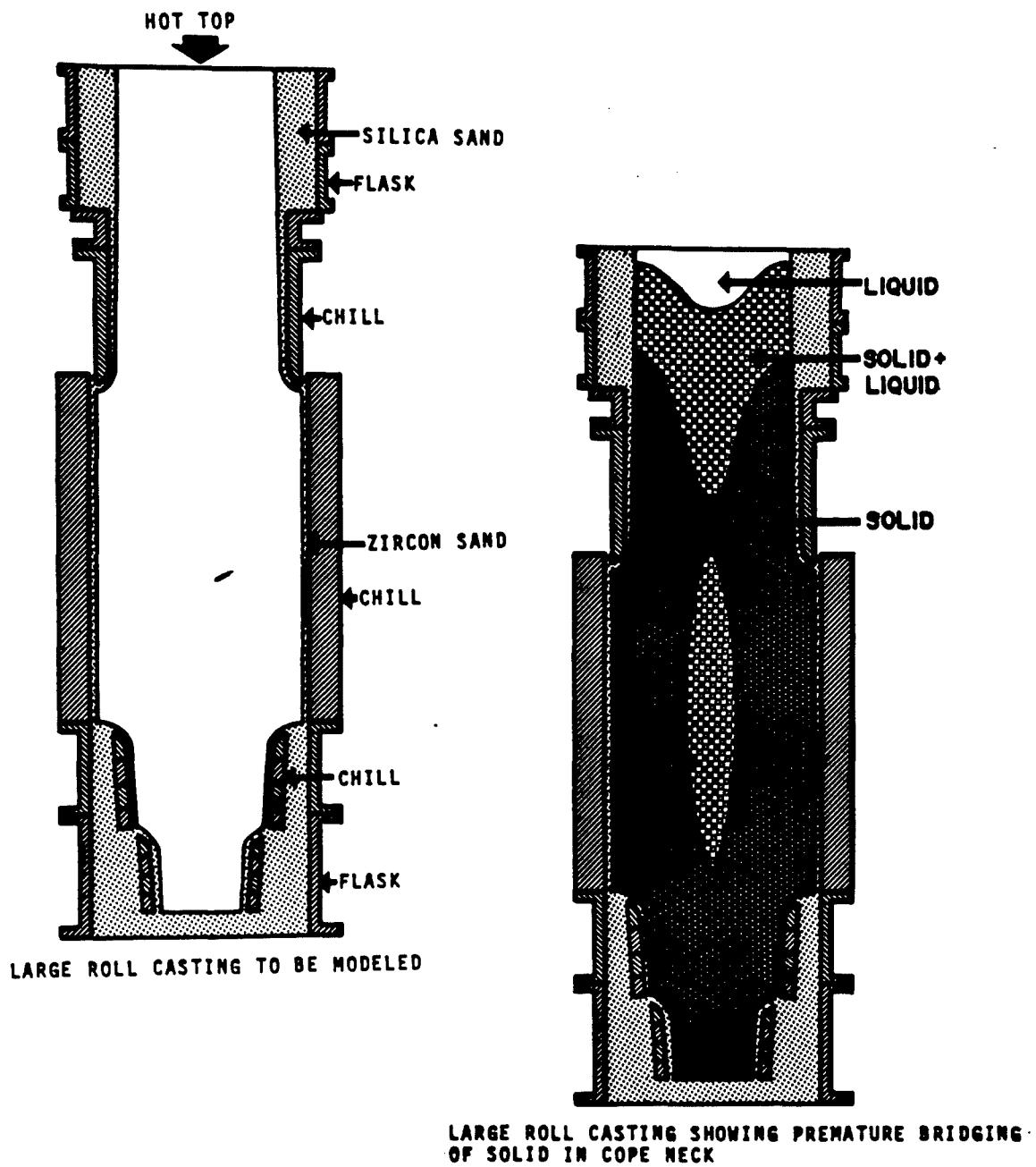


Figure 1-3: Large steel roll castings were modeled to predict bridging.

The total number of elements may be kept down by making maximum use of any symmetry which exists in the design. Each plane of symmetry obviously can cut the mesh size in half. Even if the plane of symmetry is only approximate it may allow the problem to be solved as two smaller independent problems. If the shape is axisymmetric it reduces a three-dimensional to a two-dimensional problem at tremendous savings. The shape shown in Figure I-1 contains only one plane of symmetry (parallel to the paper), whereas the shape shown in Figure I-3 contains an axis of symmetry. Figure I-3 also contains a horizontal plane of approximate symmetry in the middle of the body, permitting the top and bottom halves to be calculated separately. Large steel roll castings such as shown in Figure I-3 have been modeled successfully to predict the occurrence of premature bridging of solid in the neck which results in voids below this region. Such premature bridging is shown in the drawing on the right. Further simulations showed the modifications needed in the design to prevent this type of defect.

Mesh generation usually must be done by a person who understands the requirements of both casting design and the analytical method (FDM or FEM). This person must use intuition, experience, and rules of thumb to generate a mesh which is reasonably efficient and still an accurate representation of the actual mold and casting. There are few such people so far and this has inhibited the use of such systems. Even the commercially available mesh generation systems such as UNISTRUC do not remove the need for such a person but simply serve as aids which speed the translation of his judgments into numerical data.

It is important for the ultimate success of modeling systems for sand casting that they include improved methods for translating proposed mold and casting designs into the mesh for solidification calculations. Such systems should make maximum use of digitizers or similar devices to translate points on a drawing into numerical form. They should contain default parameters that will relieve the designer of the task of choosing the element size which is needed to give appropriate resolution.

A system designed to generate the mesh with minimum input from the user is currently being developed at Battelle-Columbus Laboratories for the University of Pittsburgh and the Army Tank Command.<sup>25</sup>

Realistic representation of a casting and mold design also involves the physical properties of the materials. Density, thermal conductivity, and specific heat are temperature-dependent properties. Latent heat of solidification for alloys is given off over a range of temperatures and the temperature dependence is not generally linear between the liquidus and solidus. How closely the simulation follows the

physical reality can have a considerable influence on the computer time required. Approximations which retain the linear nature of the heat transfer equations are highly desirable.

Our programs<sup>13</sup> are designed to accept segmented linear representations of these properties (Figure I-4). The length of the segments can be controlled by the user in some versions. This means that, for example, these properties can be represented as constants over the entire solid range or they can be tabulated at certain temperatures throughout the range and the program will perform linear interpolations for temperatures between tabulated values. Figure I-4 shows two ways of representing the heat capacity and latent heat of a solidifying metal. In the left diagram an artificially high value of  $C_p$  in the solidification range is used to account for the latent heat of fusion. In the diagram on the right a segmented approximation of the sensible heat includes the effect of temperature-dependent specific heat and a latent heat of fusion distributed over the solidification range.

### I.3.2 Flexibility

Programs can be written to be very general, but often this results in more cumbersome programs with higher costs. For example, programs which are written to handle two-dimensional mesh can be more efficient than using a three-dimensional program for the job. Axisymmetric geometry can be handled most efficiently in a two-dimensional program designed to handle this particular geometry. If average physical properties are to be used, it is best to eliminate those features in the program which are designed to accommodate their temperature dependence. Arrays should preferably be dimensioned quite close to the size that will actually be used. This means that if the simulation is written in a language like FORTRAN, it should be preceded by a utility program which operates at the monitor level to make the desired changes and selection of options before loading and compiling it. Here, too, the programs should be written so that they can be used by persons who are not experts at computer programming.

### I.3.3 Accuracy

Accuracy of results is influenced by the computational method and by the accuracy of the representation of the physical and geometric factors. Errors arising out of the computational method can frequently be evaluated by comparison with analytical solutions for simple shapes. Other errors must be evaluated by comparison with experiments.

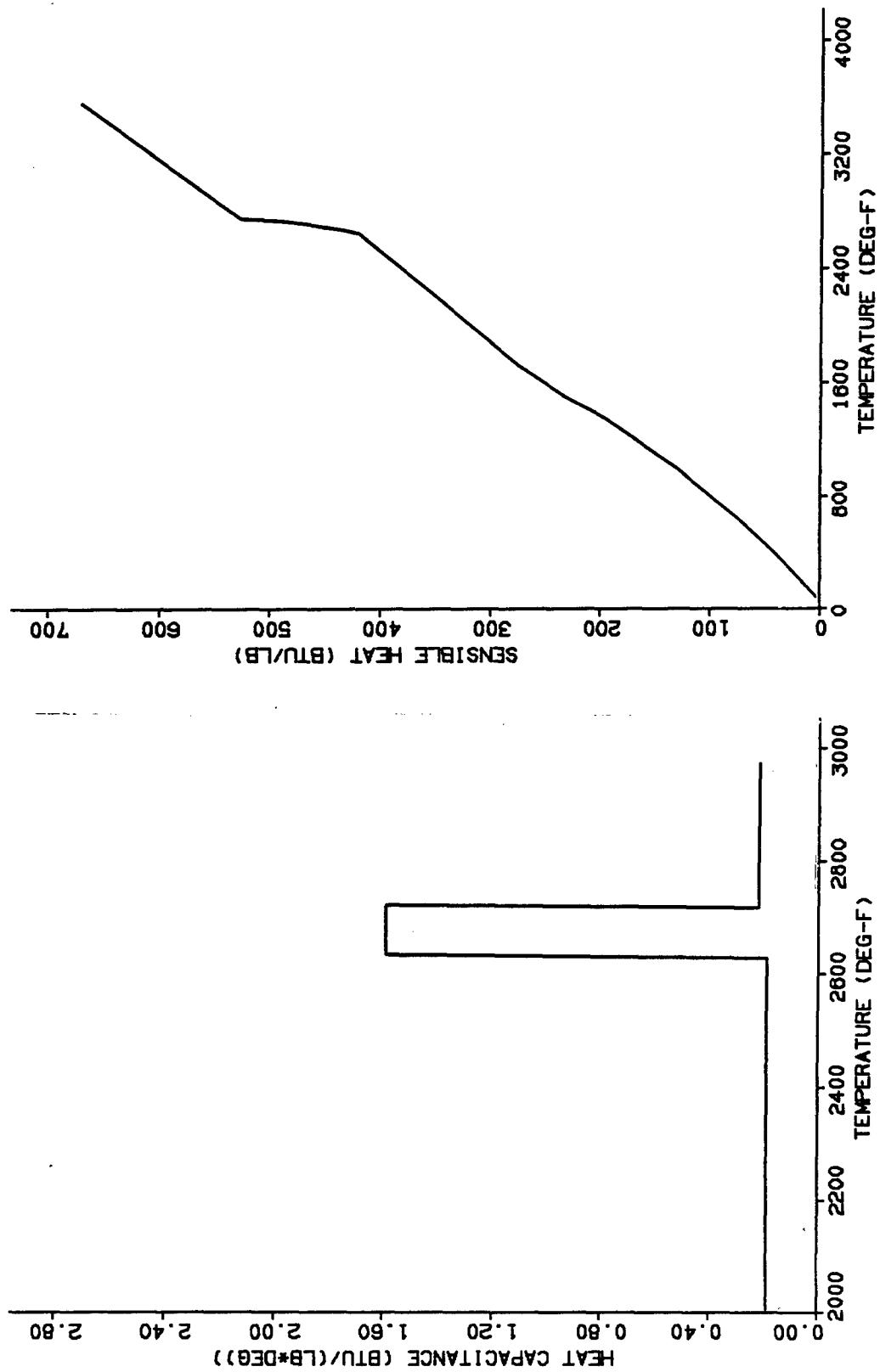


Figure I-4: Two ways of representing the heat capacity and latent heat

Figure 1-5 shows comparisons of results generated by our finite element program with results obtained by classic analytical techniques for the heating of a steel bar when one end is brought into contact with a heat source at a constant temperature. It is essential that any simulation program pass these tests, but in themselves they are not sufficient.

Agreement with experimental results is a more demanding but more nebulous objective. Many details must be known. Proper representation often requires knowledge of such things as temperature-dependent physical properties, transformation temperatures, and heat transfer coefficients with more accuracy than has been known before. The modeling program thus gives rise to the need for more experimental work.

The availability of a modeling program also provides an additional way to evaluate the physical parameters needed, so long as the effects of various variables can be separated. For example, the conductivity of molding sands could be evaluated under actual casting conditions if the other parameters could be held constant or could be accurately measured.<sup>10</sup> This separation of effects is often not easy and great care should be taken to avoid the application of compensating errors which may force the solution to conform to the experimental results but cause other solutions to be farther from the truth.

#### I.3.4 Stability

Application of either the finite difference method or the finite element method to transient heat transfer problems can introduce oscillations in temperature which are not present in nature and which are in fact prohibited by the laws of thermodynamics. To avoid these problems certain relationships must be observed between the geometric spacing ( $\Delta x$ ) and the time step ( $\Delta t$ ). In the explicit form of the finite difference equations the criterion for stability is  $\Delta t < (\Delta x)^2/(2\alpha)$  for a one-dimensional mesh (or  $\Delta t < (\Delta x)^2/(4\alpha)$  for a two-dimensional mesh, or  $\Delta t < (\Delta x)^2/(6\alpha)$  for a three dimensional mesh, where  $\alpha$  is the thermal diffusivity). In the finite element method the criterion for stability is just the reverse and a minimum time step must be specified according to the criterion  $\Delta t > (\Delta x)^2/(4\alpha)$ . This is true in at least some commercially available finite element programs<sup>26</sup> as well as one of our own.<sup>24</sup> It may arise from the use of an explicit finite difference step in the time domain along with the finite element representation in the geometric domain.

These criteria become particularly restrictive when the mesh has been drawn to

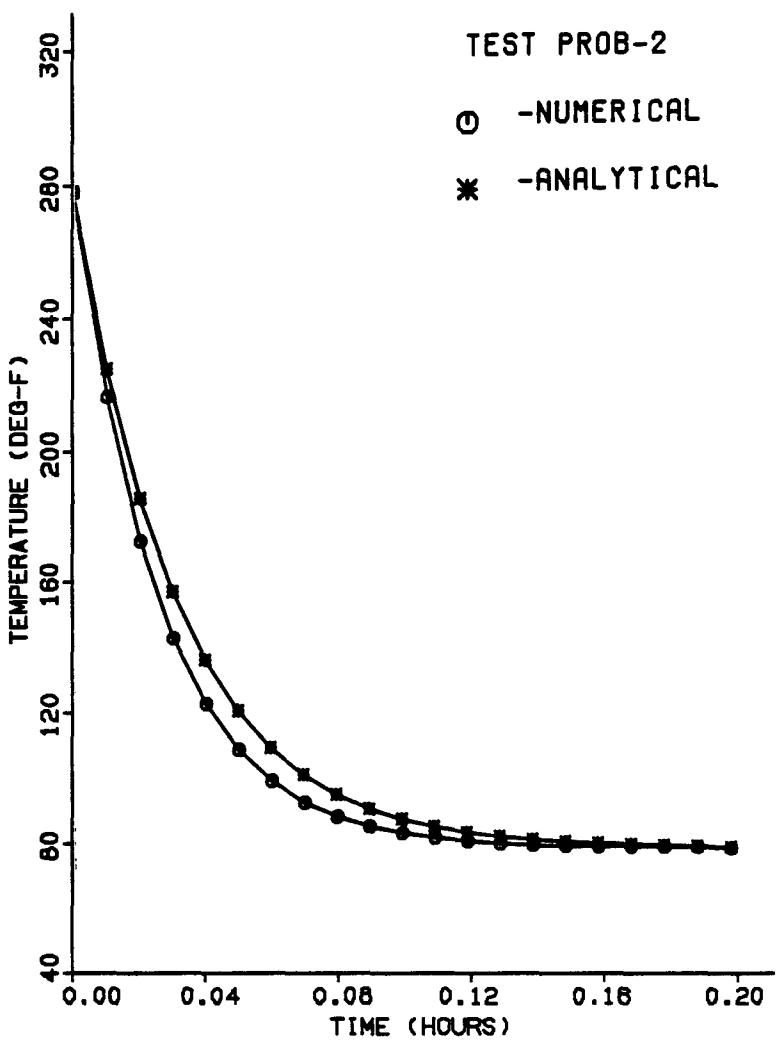


Figure 1-5: Solutions for convective cooling of a fin.

represent a mold and casting design containing several materials with widely differing  $\alpha$ 's and where the values of  $\Delta x$  have been dictated by their actual thickness in the metal.

The implicit form of the finite difference equations is inherently stable and so values of  $\Delta t$  and  $\Delta x$  may be chosen independently. A price is paid for this in two ways: (1) The heat transfer equations must be solved as a set of simultaneous equations instead of consecutively. (2) The model behaves as though it is completely damped and so it is more sluggish than the real system.

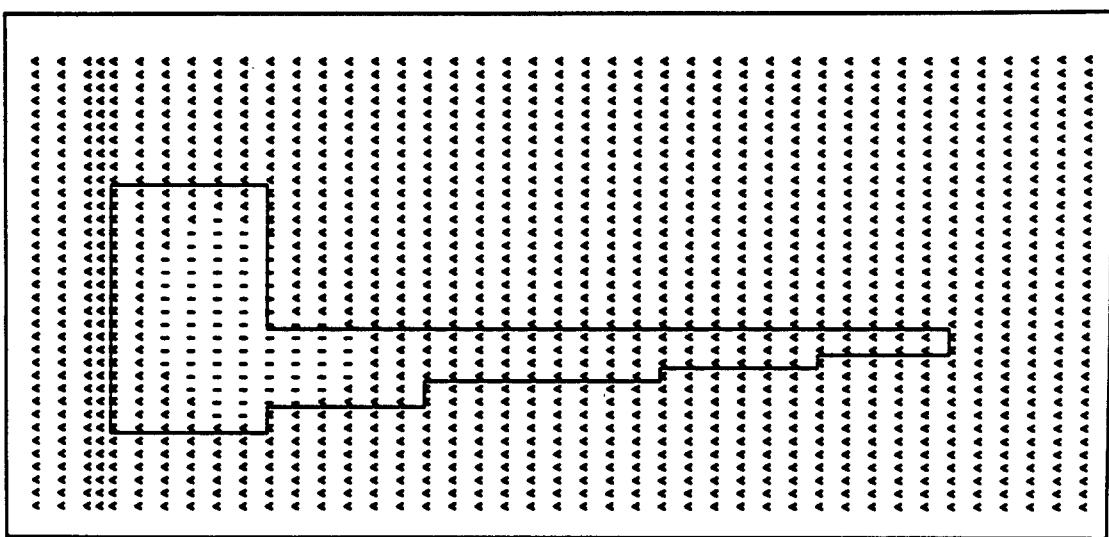
Solution of the heat transfer equations as a set of simultaneous equations is not as laborious as it might appear at first, since only the coefficients representing the heat transfer between nearest neighbor elements are needed. Thus if the system contains 1000 elements, the 1000 equations needed may be represented by two 1000 element column arrays  $T$  and  $B$  (representing the temperatures of the nodes and the constants, respectively) and one  $6 \times 1000$  element array  $A$  for the coefficients. For a two-dimensional mesh the latter would be a  $4 \times 1000$  element array. Using the Gauss-Seidel method, a very efficient routine can be written to solve this set of equations.

If very large meshes are to be used (say in excess of 3000 nodes) this technique can be modified to use disc memory so that the only part of these arrays need to be in core memory at one time.

Our experience with the implicit form of the finite difference equations indicates it has absolutely no stability problems even in regions of high temperature gradient and in regions in the mushy zone where gradients are very low. The finite element program, however, must obey the criterion stated above or instability will develop.

### I.3.5 Easily Interpreted Results

The raw results of these programs are in the form of temperatures in the nodes at each time step. This form of output is not directly useful to the designer and so post-processing is used to put it into more useful form. It is usually best to put the original output file onto tape so that it may be processed later into whatever form the user desires. Graphical outputs<sup>24</sup> include plots of isotherms and liquidus/solidus locations, cooling rates through certain temperature ranges, and temperature gradients. Note Figure I-6 in which the location of the material at a temperature between the liquidus and solidus is shown by dashes "--" and material at a temperature below the solidus is shown by a left caret "<". Plots of cooling



**Figure 1-6:** Plotter representation of solidifying plate casting.

rates such as those shown in Figure I-7 can be very important in predicting microstructures which will form in castings. Temperature gradients such as those shown in Figure I-8 can be important for predicting microstructures and stresses during solidification.

The possibility of utilizing plots produced on a line printer may provide the lowest cost method at some installations. A line printer or terminal can be used to produce graphical output as shown in the representation of a roll casting in Figure I-9 which is similar to those shown in Figure I-3. The center line of the casting is at the left. Vertical distances have been condensed much more than horizontal distances because horizontal resolution is much more important in predicting the soundness of these castings. Liquid regions are left blank. Solid plus liquid regions are represented by a period. All solid regions are illustrated by a "+". Mold material is represented by "<".

The ratio of temperature gradient to rate of liquidus or solidus advance (the G/R ratio) can be calculated and displayed as an aid to predicting both macro- and microsegregation and the microstructure which will be found in parts of the casting.

### I.3.6 Economically Justifiable Costs

All of the factors listed above contribute to the overall costs of using a casting simulation program. Computer costs may be a relatively small fraction of the overall costs. Overall costs of a simulation may run from a few hundred to a few thousand dollars, depending primarily on the complexity of the mold and casting.

These costs are justifiable if they reduce the number of trial castings or the number of bad castings which are made when a new design is produced. Some large steel castings, such as mill backup rolls, may weigh 40 to 80 tons and be worth several hundred thousand dollars each. A producer may take orders for them in quantities of one at a time. The cost of modeling may easily be justified when a new design is required. For smaller, less costly, higher production parts the lead time required to design and prove a certain design may be quite long, so that any reduction in this time can save considerable money. The costs of performing simulations may also be justified if the result is better casting designs, where "better" means designs with less conservative rigging, higher yields, lower energy and cleaning costs, and/or fewer defects and better metallurgical properties.

Foundries may find the cost of the modeling system to be attractive if the same computer and graphic installation can be used for drafting, estimating costs, quality

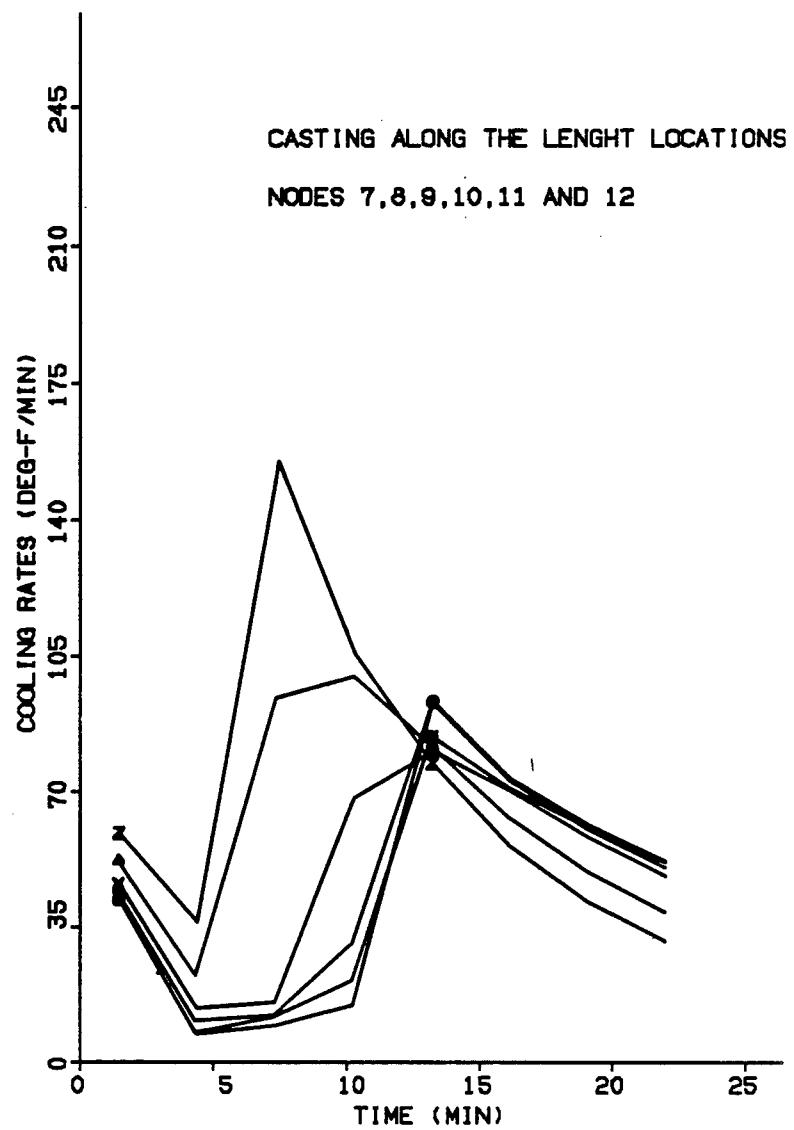


Figure 1-7: Plots of cooling rates along centerline of a plate casting.

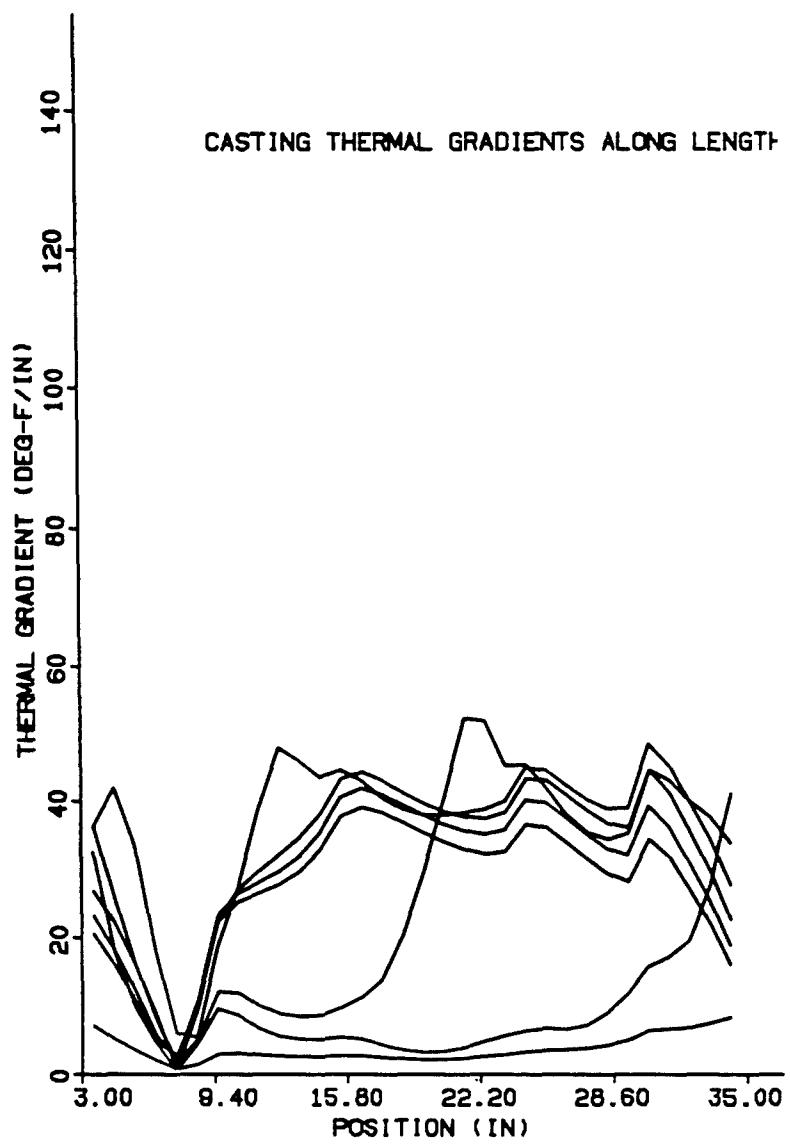
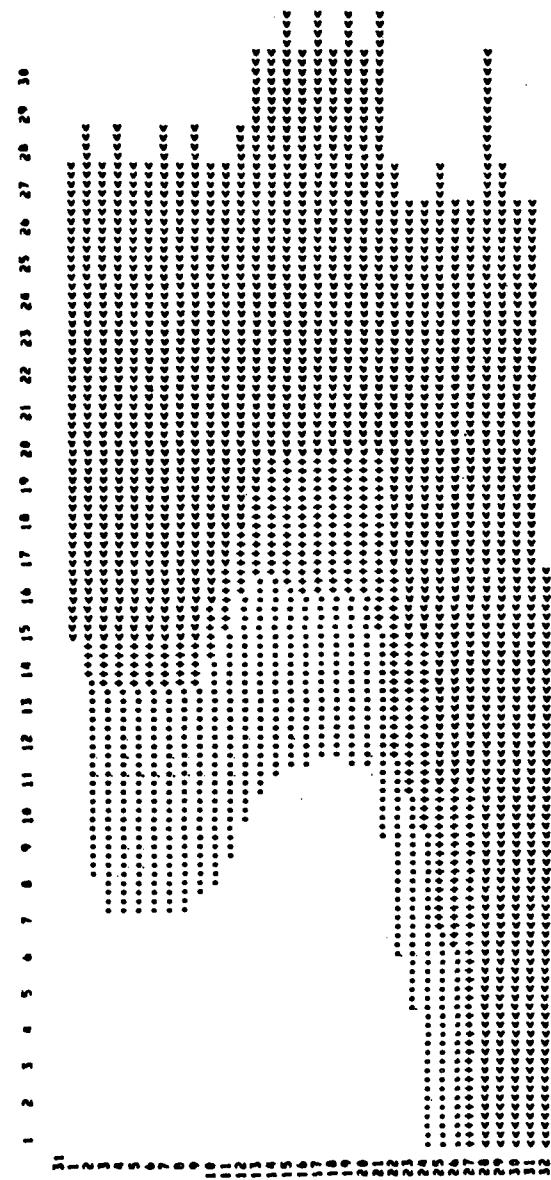


Figure 1-8: Thermal gradients along the centerline of a plate casting.



**Figure 1-9:** Line printer representation of roll solidification.

control, material tracking and record keeping, so that the total cost may be distributed over more functions.

## I.4 CONCLUSIONS

Currently available systems provide simulations of solidification of sand castings which are useful to designers in their present form. The process of making them more acceptable to the user continues to go along the lines discussed in this paper. These improvements involve both the interface with the user and the internal efficiency of the programs, both of which are necessary to expand the number of applications where the use of these models may be justified economically.

### Acknowledgements

Work at the University of Pittsburgh described in this report has been preformed primarily with financial support from the U. S. Army Tank Automotive Command, Contract No. DAAK30-78-C-107 and the Roll Manufacturers Institute, Pittsburgh, PA, Project Nos. 41, 41A, and 41B.

### References

1. Schniewind, J., "Solution of the Solidification Problem of a One-dimensional Medium by a New Method", Journal of the Iron and Steel Institute, Vol. 201 (July, 1963), pp. 394- .
2. Henzel, J.G., Jr., and Keverian, J., "The Theory and Application of Digital Computer in Predicting Solidification Patterns", Journal of Metals, Vol 17 (May, 1965), pp. 561-568.
3. Mizikar, E.A., "Mathematical Heat Transfer Model for Solidification of Continuously Cast Slabs", Trans. Met. Soc. of A.I.M.E., Vol 239 (1967), pp. 1747-1753.
4. Lait, J.E., Brimacombe, J.K., and Weinberg, F., "Mathematical Modeling of Heat Flow in Continuous Casting of Steel", Ironmaking and Steelmaking, Vol 1 (1974), pp. 90-97.
5. Sciamma, G., "Computation of Cooling Time in Permanent Mold Cast Cylindrical Bars-Checking Tests", Cast Metals Research Journal, Vol. 4 (1968), pp. 62-68.
6. Sciamma, G. and Jeancolas, M., "Temps de Solidification de Pieces Elementaires Coulees en Sable", Fonderie, Vol 303 (July, 1971), pp. 239-250.

7. Sciamia, G., "Etude la Solidification de Profils Types a Points Chauds en Fonte", Fonderie, Vol 306 (Nov., 1971), pp. 363-370.
8. Marrone, R.E., Wilkes, J.O., and Pehlke, R.D., "Numerical Simulation of Solidification", Parts I and II, Cast Metals Research Journal, Vol. 6, (Dec., 1970), pp. 184-192.
9. Pehlke, R.D., Kirt, M.J., Marrone, R.E., and Cook, C.J., "Numerical Simulation of Casting Solidification", Cast Metals Research Journal, Vol. 9 (June, 1973), pp. 49-55.
10. Kirt, M.J., and Pehlke, R.D., "Determination of Material Thermal Properties Using Computer Techniques", Transactions of American Foundrymen's Society, Vol. 81 (1973), pp. 524-528.
11. Jeyarajan, A., and Pehlke, R.D., "Computer Simulation of Casting with a Chill", Transactions of American Foundrymen's Society, Vol. 84 (1976), pp. 647-652.
12. Stoehr, R.A., and Brody, H.D., Solidification Contours of Selected Rolls, The Roll Manufacturers' Institute, Pittsburgh, PA. (1976).
13. Brody, H.D. and Stoehr, R.A., "Computer Simulation of Heat Flow in Casting", Journal of Metals, Vol. 32, No. 9 (Sept., 1980), pp. 20-27.
14. Das, P.K., "Finite Element Analysis Method in Design of Castings", Transactions of American Foundrymen's Society, Vol. 13 (1979), pp. 597-600.
15. Orivuori, S., "Efficient Method for Solution of Nonlinear Heat Conduction Problems", International Journal for Numerical Methods in Engineering, Vol. 14 (1979), pp. 1461-1476.
16. Brunch, J.C. and Zyvoloski, G., "Transient Two-dimensional Heat Conduction Problems Solved by the Finite Element Method", International Journal for Numerical Methods in Engineering, Vol. 8 (1974), pp. 481-494.
17. Comini, G., Del Guidice, S., Lewis, R.W., and Zienkiewicz, O.C., "Finite Element Solution on Non-linear Heat Conduction Problems with Special Reference to Phase Change", International Journal for Numerical Methods in Engineering, Vol. 8 (1974), pp. 613-624.
18. Morgan, K., Lewis, R.W., and Zienkiewicz, O.C., "An Improved Algorithm for

Heat Conduction with Phase Change", International Journal for Numerical Methods in Engineering, Vol. 12 (1978), pp. 1191-1195.

19. Donea, J., "On the Accuracy of Finite Element Solutions to the Transient Heat Conducting Equation", International Journal for Numerical Methods in Engineering, Vol. 8 (1974), pp. 103-110.

20. Celia, A., "On the Accuracy and Stability of the Finite Element Approximation for Parabolic and Hyperbolic Operation", Conf. on Mathematics of Finite Elements and Applications, Brunel University, 1975 (New York: Academic Press Inc. 1976), pp. 183-189.

21. Mathew, J. and Brody, H.D., "Analysis of Heat Transfer in Continuous Casting Using Finite Element Method", Computer Simulation for Materials Applications, Nuclear Metallurgy, R.J. Arsenault, et al. (eds.), Vol. 20, Part 2 (1976), pp. 1138-1150.

22. Mathew, J. and Brody, H.D., "Simulation of Thermal Stresses in Continuous Casting Using a Finite Element Method", Computer Simulation for Materials Applications, Nuclear Metallurgy, R.J. Arsenault, et al. (eds.), Vol. 20, Part 2 (1976), pp. 978-990.

23. Mathew, J. and Brody, H.D., "Simulation of Heat Flow and Thermal Stress in Axisymmetric Continuous Casting", Solidification and Casting of Metals, The Metals Society (London, 1979), pp. 244-249.

24. Schwartz, W., Simulation of Heat Flow During Solidification, M.S. Thesis, University of Pittsburgh (1980).

25. Badawy, A., Schreiber, K., and Akgerman, N., "CSTMS3, A Finite Element Mesh Generator for Castings", Modeling of Casting and Welding Processes, eds. H. D. Brody and D. Apelian, T.M.S.-A.I.M.E. (Warrendale, Pa., 1981).

26. ANSYS User Information Manual, Control Data Corporation (1978).

## II. A FINITE ELEMENT MESH GENERATOR FOR CASTINGS

A. Badawy and K. Schreiber  
Battelle's Columbus Laboratories  
Columbus, Ohio

J. Chevalier and T. Wessel  
U.S. Army Tank Automotive Command  
Warren, Michigan

### SUMMARY

An automatic finite-element mesh generation program (CSTMS3\*\*) for casting models is described. The program is designed to be used by foundry engineers with no previous programming or finite-element experience. The output of CSTMS3 is to be used later in the analysis of mold heat flow, fluid flow, and stress analyses to provide for castings soundness.

\* This paper was originally presented and published in the symposium proceedings: Modeling of Welding and Casting Processes, ed. H. D. Brody and D. Apelian (T.M.S.-A.I.M.E., New York, 1981).

\*\* An updated version of CSTMS3 has been incorporated in subprogram MGEN of the UPCAST software.

## II.1 INTRODUCTION

The development of the finite-element method provides engineers with a general and powerful tool for the analysis of continuum mechanics problems. A general problem as a whole may be formulated by the finite-element method through the following procedure:

- Discretization of the continuum (to finite elements).
- Selection of shape functions (within each element).
- Obtaining the element characteristic matrix.
- Assembly of the algebraic equations (this includes the assembly of the element matrices into the overall master matrix of the discretized continuum as a whole).
- Solution of the algebraic equations for the unknown parameters.

The theory behind each of the previous steps is well explained in many publications (1, 2, 3,...etc.). This paper deals only with the first topic, the discretization of the continuum or what is usually called finite element mesh generation.

## II.2 GENENERATION OF MESH DATA

As mentioned before, the first step in a finite-element analysis is to divide the continuum (by imaginary lines or surfaces) into a number of finite elements. This process is still essentially a matter of engineering judgment to decide what number, size and arrangement of finite elements that give an effective representation of a particular continuum. Ideally, a finite element computer program should generate its own mesh data from a minimum number of geometric parameters. The amount of input data required is minimal and once the relevant coding has been written and tested, the possibility of errors is largely eliminated.

Program CSTMS3, described herein, is an automatic mesh generator for three-dimensional finite element meshes composed of brick (8 node) and wedge (6 node) elements. CSTMS3 constructs a mesh by assembling a number of user defined zones (blocks), each of which is automatically subdivided into finite elements. The program can handle casting models composed of parallelopipeds and cylindrical zones. Other zone shapes are currently tested (hollow cylinders, cones, hollow cones, etc.).

CSTMS3 is designed to be used by casting engineers with no previous programming or finite-element experience. The output of CSTMS3 is to be used later in the analysis of mold heat flow, fluid flow, and stress analysis to prove for castings soundness. Interactive design (man-machine and machine-man) of casting molds is another important output of CSTMS3. In this paper, however, we explain only the capabilities of CSTMS3 to generate 3D mesh data.

Since the main goal is to study the heat transfer between the molten metal of the casting model and the surrounding sand (sand mold), it is clearly advantageous to have a finer mesh in the regions describing the contact between the molten metal and the sand.\* For that reason, program CSTMS3 automatically enlarges and reduces the original input geometry. The rate of enlargement and reduction decreases with distance from the contact surface (between the metal and sand) inwards (metal) and outwards (sand). To assist in understanding the capabilities and advantages of using program CSTMS3, Figure II-1 will be used as the basic input geometry. To generate the mesh the user should follow the steps outlined in the subsections that follow.

### II.2.1 Prepare the Geometrical Data

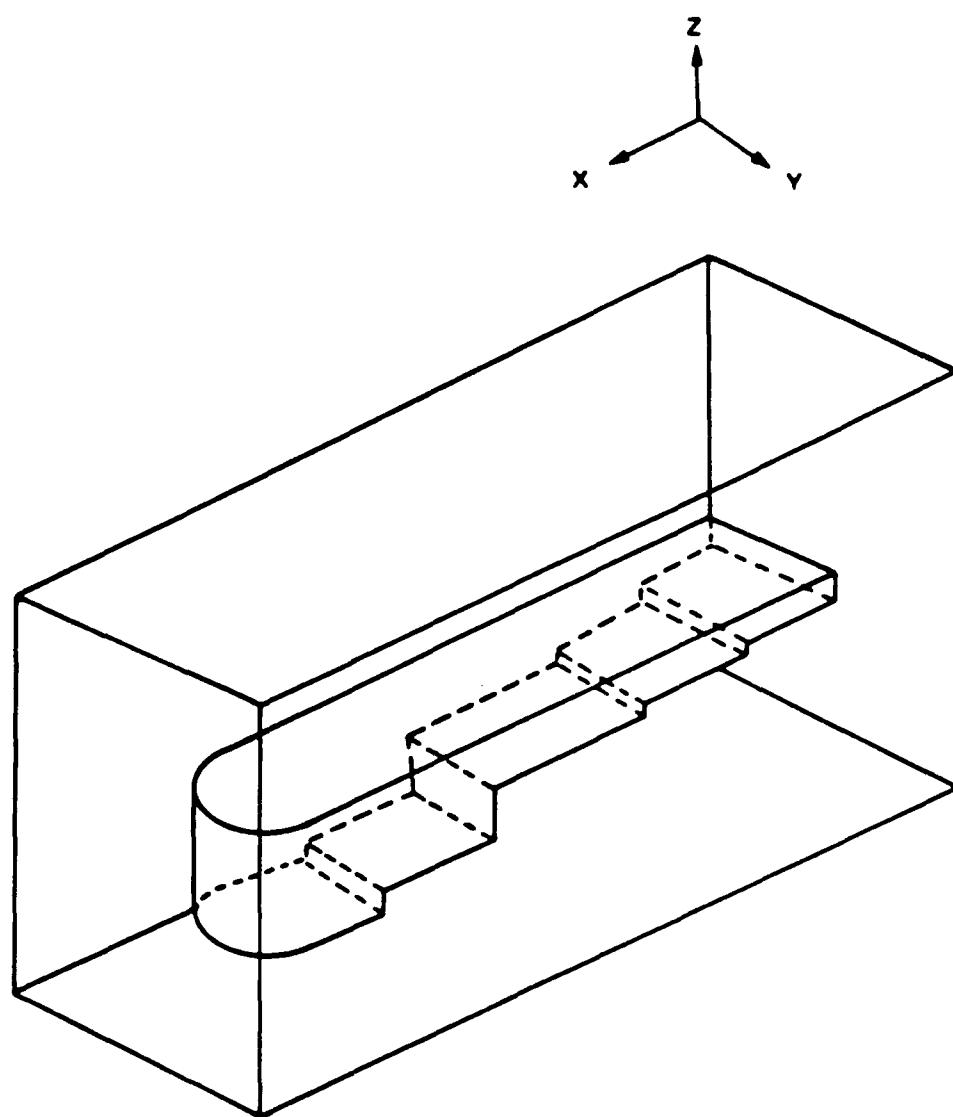
To define the geometry of a casting and mold configuration as in Figure II-1, the user should proceed as outlined below.

1. Divide the model into a certain number of "zones". A zone is the basic building block for the model. To create a model, the user must visualize a subdivision of his casting into zones in a manner similar to that used to manually create a finite element mesh. So far, only two zone shapes are available to the user, "brick" and "cylindrical". A brick zone is a volume bounded by six planes. A cylindrical zone is a volume bounded by a cylinder. Figure II-2 shows a possible way of dividing the model (five brick zones, including the sand box, and one cylindrical zone).
2. Prepare data defining the zones. Zones 1 to 4 (brick zones) should be defined using eight X, Y, Z coordinates; e.g., based on Figure II-1, Zone 6 (sand mold) would be defined by:

0.0	0.0	0.0
34.0,	0.0,	0.0
34.0,	14.0,	0.0

---

\*Because the highest temperature gradients in the type of castings being modeled will be found in the sand at the interface with the casting, Professor H. Brody suggested that the finest mesh divisions should be located in that region. That strategy was incorporated into the logic of CSTMS3 and MGEN.



**Figure II-1:** Test casting model.

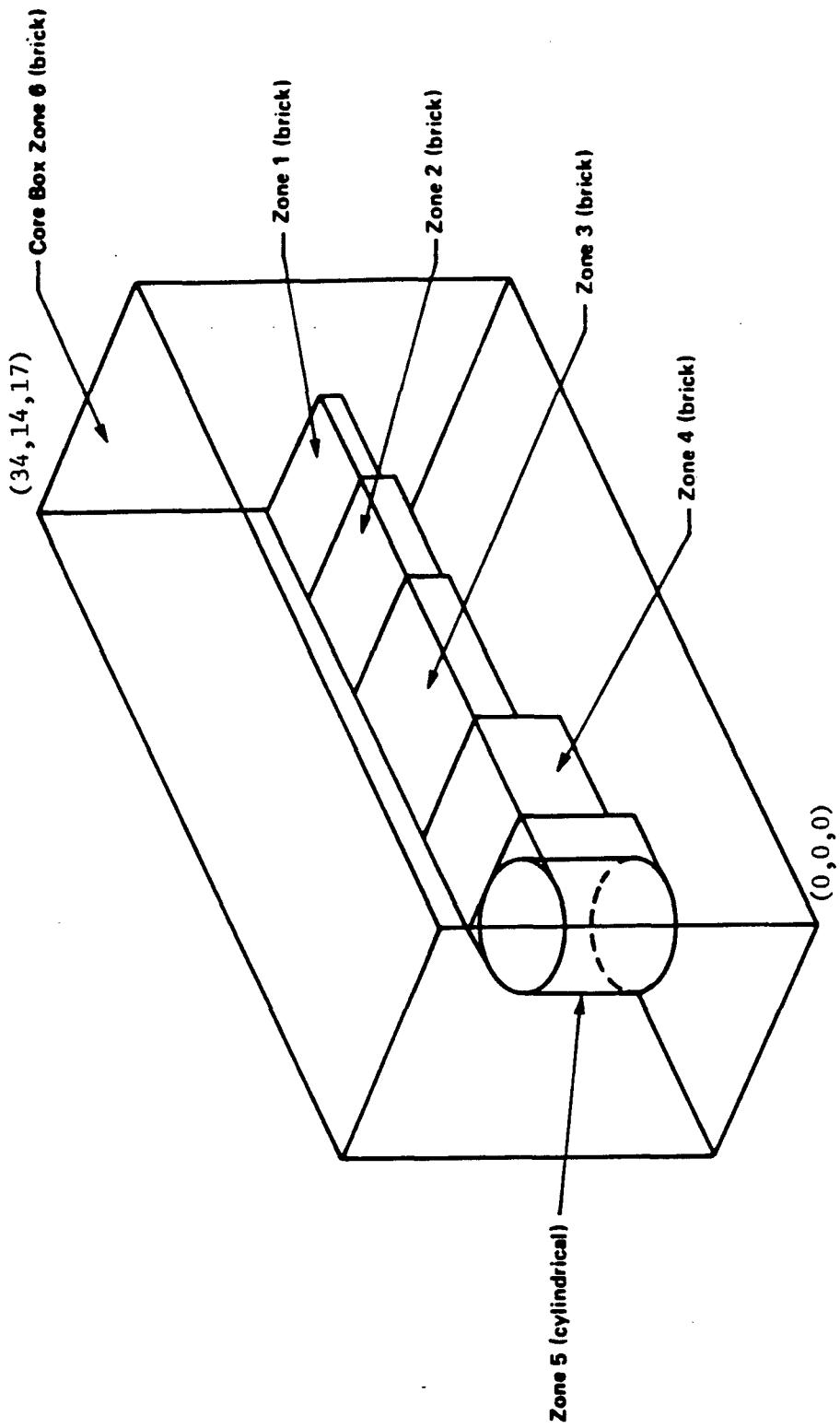


Figure II-2: Division of test casting model into zones.

0.0, 14.0, 0.0  
0.0, 0.0, 17.0  
34.0, 0.0, 17.0  
34.0, 14.0, 17.0  
0.0, 14.0, 17.0

Zone 5, the cylindrical zone, should be defined by X, Y, Z coordinates of the lower center, radius, height, and axial direction of the cylinder, e.g.,

7.5, 7.5, 3.75, 3.5, Z-direction

3. Choose a working view (W.V.), e.g., X-Y (plan), X-Z (elevation), or Y-Z (side view), and then choose an aiding view (A.V.). The idea here is for the program to use the (W.V.) to generate a 2-D mesh and then use the (A.V.) to generate the third dimension of the mesh by simply "slicing" the (A.V.) to a number of "layers". From Figure II-2 it is most convenient to use XY as a (W.V.) and Y-Z as an (A.V.).

Steps 1 and 2 should be prepared on a piece of paper before running the program. The order of defining the zones to the program does not have to follow a specific sequence. The user does not have to define the volumes connecting the cylindrical zone to its neighboring brick zones. The program automatically defines that interconnection.

### II.2.2 Input the Data to the Computer Program CSTMS3

Having prepared the geometrical data, the user is now ready to enter the program, using the following steps:

1 - Start up the computer (PDP/11 minicomputer)

2 - Run the program by entering the underlined material below. The computer will answer in the fashion indicated by the nonunderlined upper case letters.

RU CSTMS3 <CR>

WELCOME TO PROGRAM CSTMS3, FOR AUTOMATIC MESH GENERATION.

PLEASE DEFINE THE W.V.

W.V. IS X-Y <CR>

PLEASE DEFINE THE A.V.

A.V. IS Y-Z <CR>

PROGRAM IS READY TO ACCEPT GEOMETRICAL INPUT DATA.

At this stage the program is ready to accept the input data defining the casting model. The program will proceed:

DO YOU HAVE A MASTER FILE? (Y/N)

The program is asking if you have a file containing all the geometry information defining the model. After every run the program will automatically generate a master file for subsequent use. If user enters,

Y <CR>

the program will respond

ENTER THE NAME OF MASTER FILE

User should now enter the name

TEST .001 <CR>

If user does not have, or want to create a new master file he should enter

N <CR>

ENTER A NAME FOR THE MASTER FILE TO BE GENERATED

TEST .001 <CR>

READY FOR DATA DEFINITION

0-CONE, 1-BRICK, 2-CYLINDER, 3-FLASH

YOUR CHOICE

The user should choose from the number one to three, e.g. for the test model of Figure II-2:

1 <CR>

BRICK ZONE NUMBER [1]

The number between brackets is a default value; if the user wants to use it he simply hits <CR>; if he does not, he can enter any other brick zone number.

<CR>

ENTRY OF DATA FROM A FILE OR KEYBOARD [F/K]

(The X, Y, Z coordinates, defining the different zones, could be entered to the program via keyboard or through a predefined data file.) If the user enters

F <CR>

program proceeds

ENTER NAME OF DATA FILE

e.g.,

ZONE1.DAT <CR>

If the user enters

K <CR>

program proceeds

ENTER X, Y, Z, COORDINATES DEFINING ZONE

NUMBER 1.

The user then enters eight X, Y, Z coordinates to define Brick Zone 1 using the coordinates listed in preparation step 2 above. The user finishes defining all the zones (brick, cylinders, and sand mold) then enters

(0) <CR>

to indicate DONE option. The program will immediately display a "wire frame" diagram for the different zones on the CRT graphics terminal, as shown in Figure II-3.

The program then proceeds:

ARE YOU SATISFIED WITH YOUR MODEL?

This question actually indicates the beginning of the "Design Session", i.e., updating the model, modifying, deleting adding, rotating, scaling, . . . etc.

If the user is satisfied with his model, he simply enters

YES <CR>

If the user wants to change any geometrical data (interactively), he enters

NO <CR>

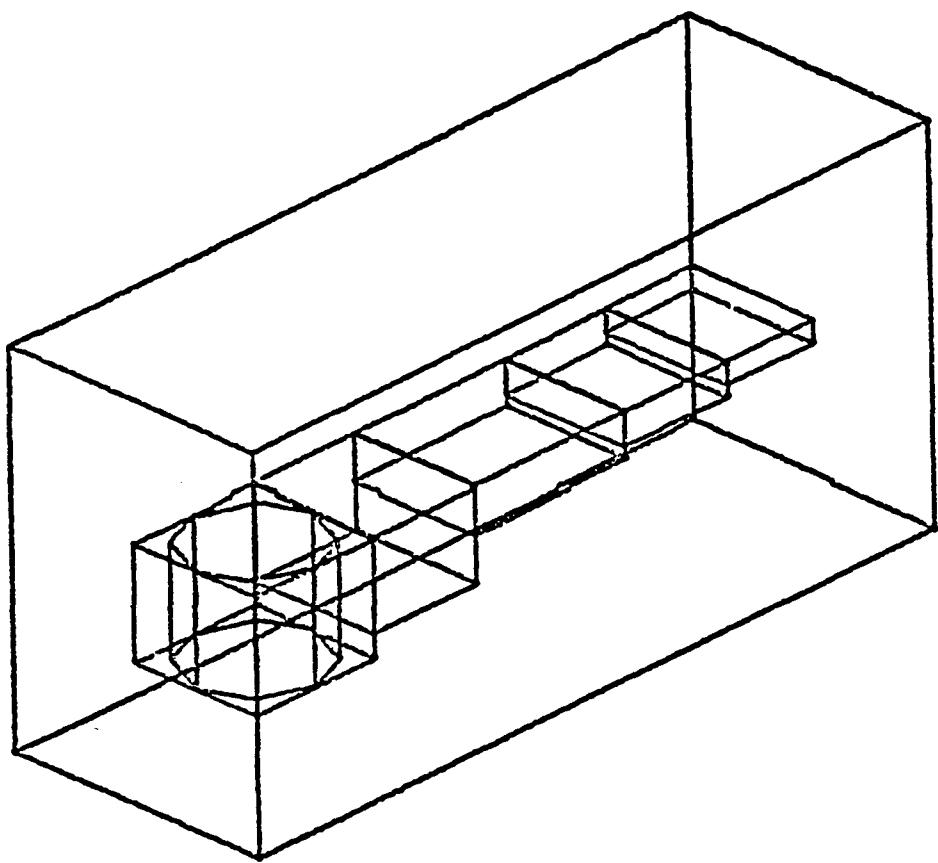
The program then asks for a change command

COMMAND -

User enters (in the same line) his modification command, e.g.,

COMMAND - DELETE <CR>

POSITION THE LIGHT PEN AT ANY PART OF THE ZONE TO BE DELETED



**Figure II-3:** CRT representation of zone model.

The user is required to hit any part of the zone he wants to delete. Program immediately deletes the hit zone. Other design commands are available to the user, e.g.,

- ROTATE for rotating any zone or the whole model through a specified angle about a specified axis.
- SCALE for scaling any particular zone or the whole model along the X, Y, or Z directions, or overall scaling.

Figure II-4 shows the effect of scaling and rotating some of the zones of Figure II-3.

After the user is done with shape and modification, he can enter

DONE <CR>

The program will then proceed as follows:

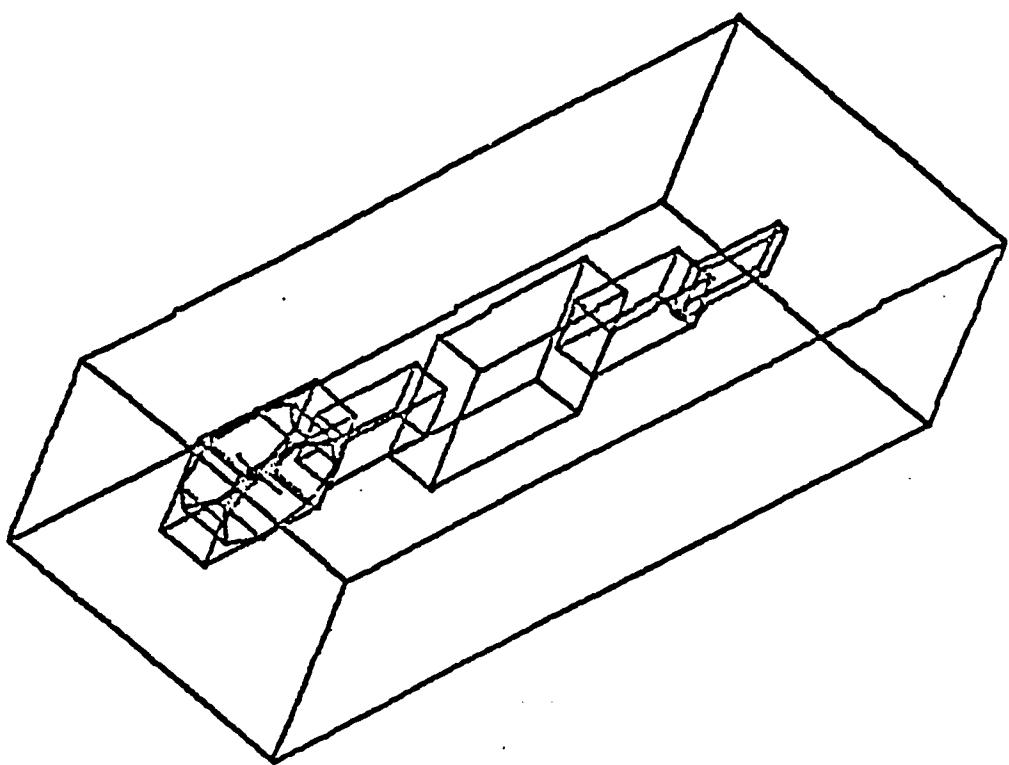
DO YOU WANT TO GENERATE THE MESH?

If the user does not want to generate the mesh, he enters

NO <CR>

The program responds by

ARE YOU SURE?



**Figure II-4:** Scaling and rotation of the zone model.

User enters

YES <CR>

Program stops at this point. Now, if the user wants to generate the mesh, he enters

YES <CR>

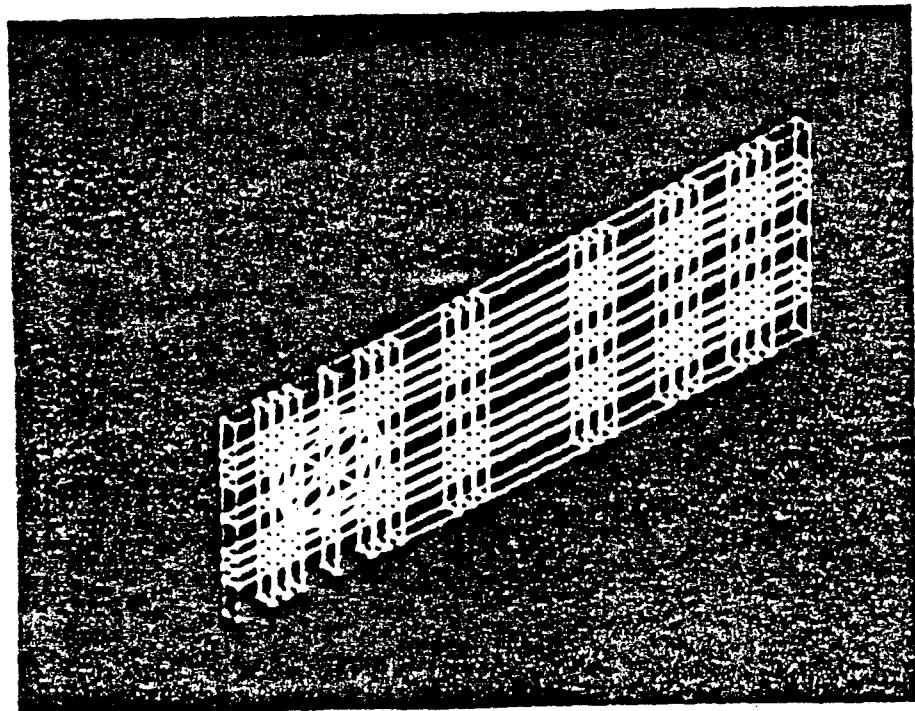
The program then asks for an output file name into which to load the generated mesh

ENTER NAME OF OUTPUT FILE

User enters any name, e.g.,

OTFIL .001 <CR>

The program will automatically generate the mesh without any interaction from the user. The program will identify metallic and sand elements and wedge and brick elements, number the nodes of each element, and generate the X, Y, Z coordinates of each node for every layer. Figure II-5 shows a 3-D plot for one of the layers as shown on the CRT terminal.



**Figure II-5:** 3-D plot for layer number 3.

## REFERENCES

- (1) O. C. Zienkiewicz, The Finite Element Method in Engineering Science, McGraw Hill, 1971.
- (2) R. H. Gallagher, Finite Element Analysis Fundamentals, Prentice Hall, 1975.
- (3) O. E. Hansteen, "Finite Element Methods as Applications of Variational Principles", Finite Element Methods in Stress Analysis, I. Holand and K. Bell, TAPIR, 1969, pp 451-473.

## ACKNOWLEDGEMENT

This study, part of an overall CAD-CAM process for designing molds, is based on a project sponsored by the U.S. Army Tank Automotive Research and Development Command, Warren, Michigan, under Contract DAAK30-78-0020. The authors wish to thank the CAD-CAM program engineer, Dr. H. Brody (University of Pittsburgh), and Dr. N. Akgerman (Battelle program manager) for their review and constructive suggestions.

### III. PROGRESS ON THE CAD/CAM CASTING PROCESS

R. Nariman and S. R. Miller  
Blaw Knox Foundry and Mill Machinery Co.

H. D. Brody, V. Srinath, W. R. Schwartz and R. A. Stoehr  
University of Pittsburgh

N. Akgeman, A. Badawy, and G. H. Wilson  
Battelle Columbus Laboratories

#### ABSTRACT

The CAD/CAM Casting Process is being developed as an aid to casting engineers in designing better steel castings in a fraction of the time presently required. The computer routines incorporate drafting routines and computer assisted computations relative to heading and gating. Evaluation routines allow 2-D and 3-D heat flow analyses with a minimum of interaction from the user. The objectives and the present stage of development are described herein.

\* This paper was presented at the conference and was published in the proceedings of the Technical and Operating Meeting, Steel Founders Society of America, Chicago, November, 1981.

### III.1 INTRODUCTION

This paper discusses Phase I of the development of a system for the planning of armor steel casting with the aid of computers. A system of programs, CAD/CAM Casting Process,\* is being developed by the University of Pittsburgh, Blaw Knox and the Battelle Columbus Laboratories under contract to the U.S. Army, Tank and Automotive Research and Development Command (TACOM). Although the programs being developed for TACOM are intended for armor steel castings, the computer routines will be applicable for steel castings in general.

### III.2 OBJECTIVES

The CAD/CAM Casting Process is aimed at providing a computer aided design tool to the casting engineer. It represents an integrated set of programs covering the two following areas:

- A framework for utilizing computer graphics to handle and display cross sectional geometry as well as calculate geometric parameters relative to heading and gating.
- Evaluation of mold cavity design by finite element analysis of the solidification process. Facilities are provided for defining a simplified three dimensional mathematical model of the casting. Actual mesh generation for the analysis is completely automatic. Output in the form of heat transfer plots and solidification maps will serve to predict casting soundness.

The overall design of the CAD/CAM Casting Process is illustrated in Figure III-1. It is intended that the programs will be used by casting engineers who do not have detailed understanding of computer techniques or finite element analysis. The casting engineer will have the facility to make accurate analyses of the freezing process reserving for himself the decisions relative to heading and gating and having the program automatically, inexpensively, and rapidly handle the details of the analysis. Facilities will be available for casting engineers with a strong background in computer analyses to override the computer routines in guiding the analysis.

---

\*The programs were developed into UPCAST.

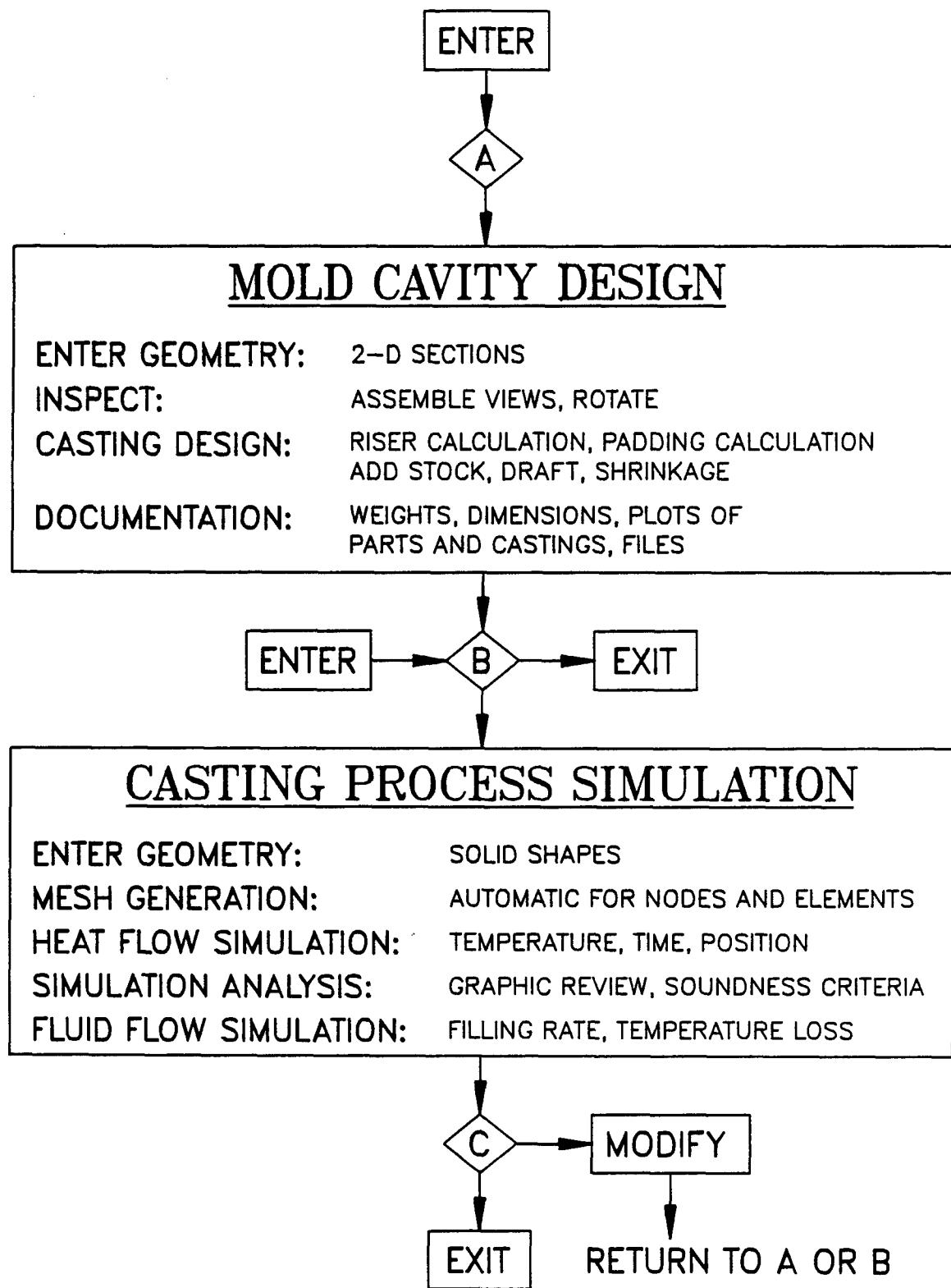


Figure III-1: Overall design of the CAD/CAM casting process.

### III.3 DESCRIPTION OF THE CAD/CAM CASTING PROCESS

The mold cavity design routines are primarily a drafting tool that make use of two input devices and a graphics screen. The input devices are a function keyboard and a digitizing tablet. With the keyboard, coordinates of points describing the perimeter of a cross-section can be input. Twenty-five points per cross-section can be used. With the use of different functions keys, the points can be joined by straight lines, arcs or smooth curves. The cross-section is then viewed on the graphics screen. There are a total of 16 main function keys. Descriptions of some of the functions follow:

1. Activate the cross-section digitizing subsystem. The user can enter new sections via the digitizer tablet.
2. Activate the display management subsystem. Display orientation, display size, and number of displayed entities can be changed.
3. Edit Sections. This function provides the facilities for graphical editing of cross-sectional data. Subfunctions such as moving, scaling, rotation, copying, mirroring are available.
4. Activate the subsystem for performing casting process design calculations on cross sections.
5. Activate library search subfunction. This will enable designers to search through existing designs for cross-sections similar to the one being worked on. In addition, frequently used shapes can be stored and recalled to minimize time and effort required to define a new geometry.
6. The contents of the graphics screen is copied to the x-ray plotter.

Figure III-2 is an example of cross-section data input via the function keyboard.

With the digitizer tablet we can input cross-sections by presenting points on the tablet with a light pen. Sections or views of a scaled blue print can, in that way, be "traced" onto the screen.

Several subfunction keys aid in this process. For instance, a circle can be traced by digitizing the center and one point of the circumference, an arc of a circle by digitizing three points, and a smooth curve by digitizing several points.

The sections on the screen can be enlarged or reduced to any desired scale and shrinkage allowances added.

---

1. EXAMPLE DATA SET IN FORMAT TYPE 1

0, 0.0, 0.0, 1.0, 0.0  
-1.5, 0.2, 0.0, 0.0  
-1.4, 1.8, 0.0, .250  
-0.9, 1.4, 0.0, .250  
-0.9, 0.6, 0.0, .500  
1.0, 0.6, 0.0, .375  
1.0, 1.2, 0.0, .375  
2.2, 1.2, 0.0, .375  
2.3, 0.2, 0.0, 0.0  
2.2, -1.0, 0.0, .375  
-1.4, -1.0, 0.0, .375  
-1.5, 0.2, 0.0, 0.0

---

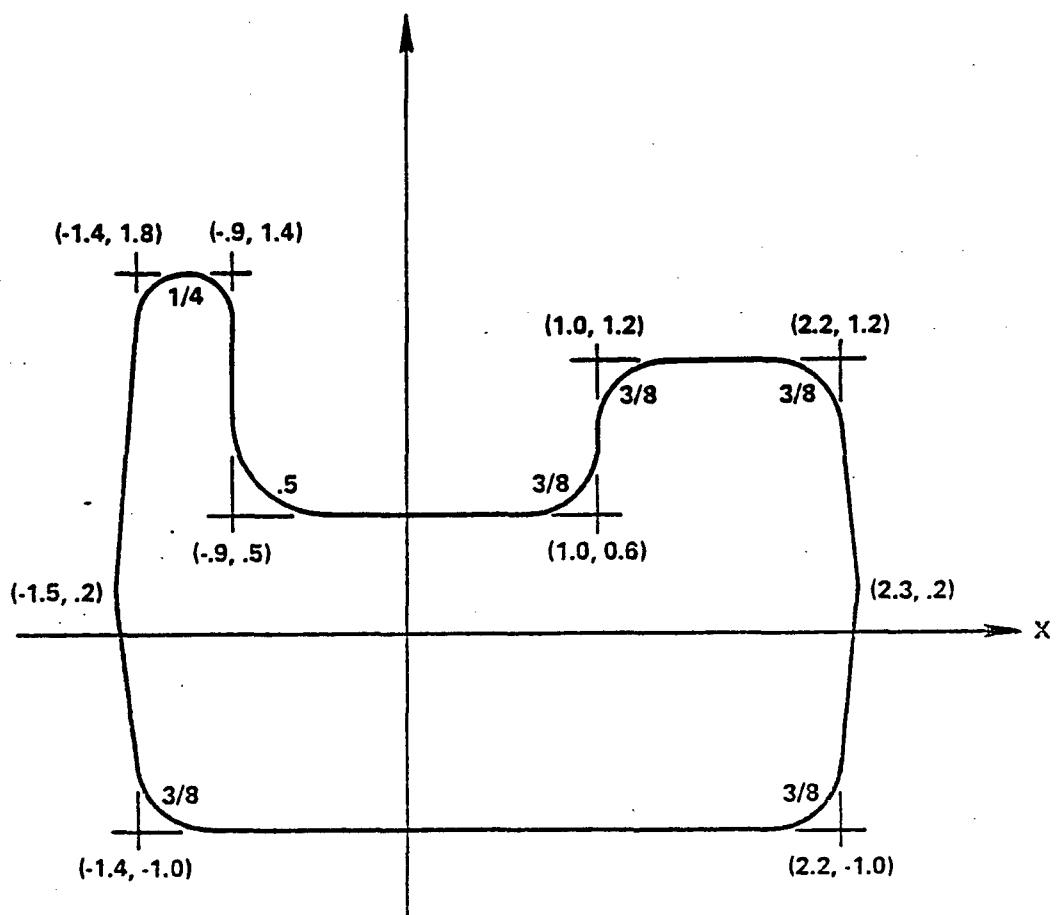


Figure III-2: Example of cross-section data input

When all the cross-sections defining a casting have been fed to the computer, the computer can be ordered to display a three dimensional, isometric view of the casting. This 3-D view can be rotated and seen from different angles.

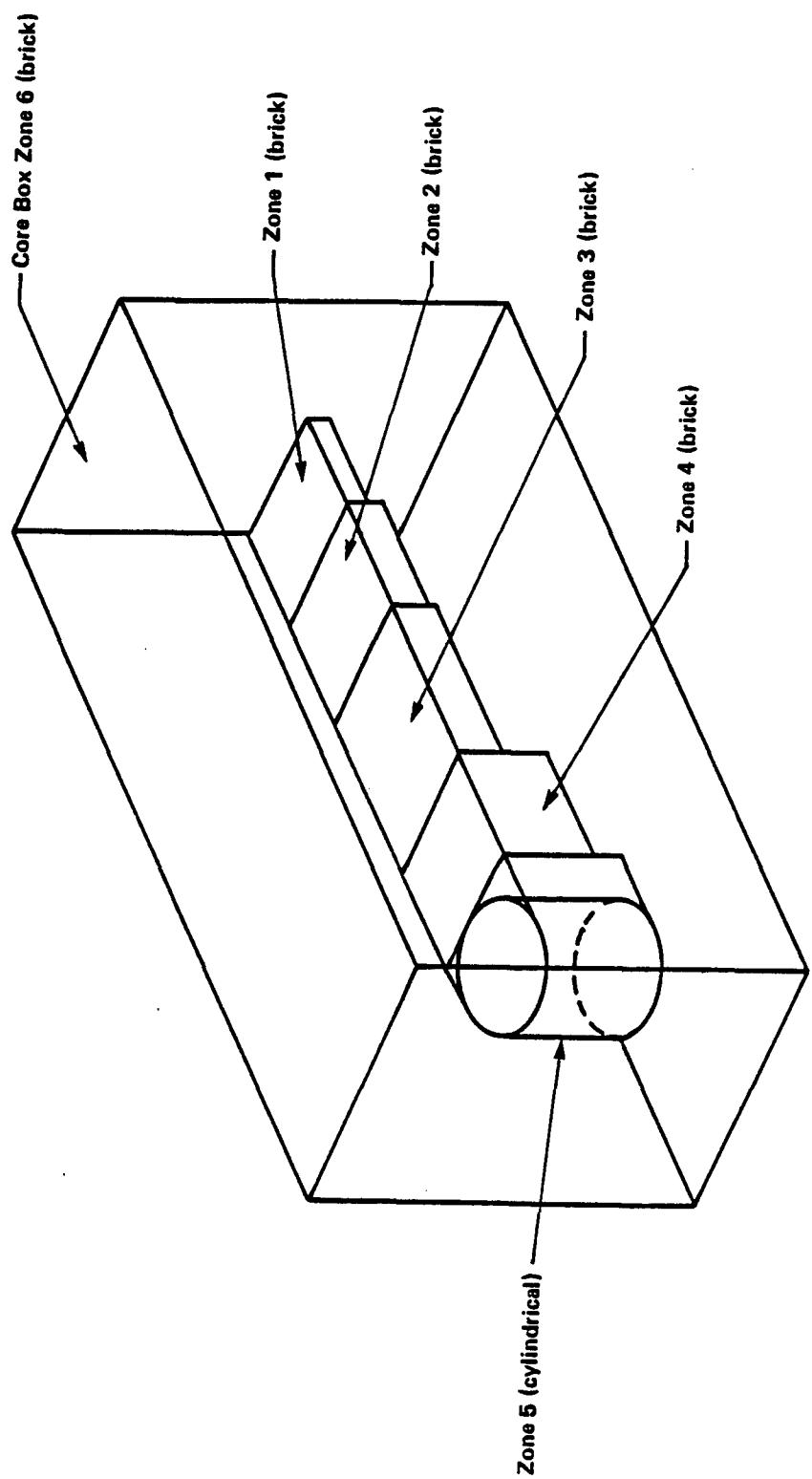
At this time, the casting engineer is ready to interact with the computer by adding machining allowances, back-ups, tapers, fillets, etc. again using the graphics terminal.

The computer can be ordered to inscribe circles along a cross-section and to display the diameters of these circles. It also calculates surface areas and perimeters.

Then the casting engineer locates the calculated feeder pads, risers, chills, gating, etc. with the aid of the graphics terminal. Hard copies of the design are then automatically printed.

To evaluate the mold cavity design routines are available to perform a heat flow simulation. The geometry of the mold cavity must be again described to the computer, this time in terms of simple geometrical shapes called zones. The shapes to be implemented in Phase I include plates (bricks), cylinders, truncated cones, hollow cylinders, and hollow cones. Figure III-3 illustrates the division into five zones of the drag of the mold for a test plate casting used in the experimental portion of the project plus one additional zone to represent the sand in the core box.

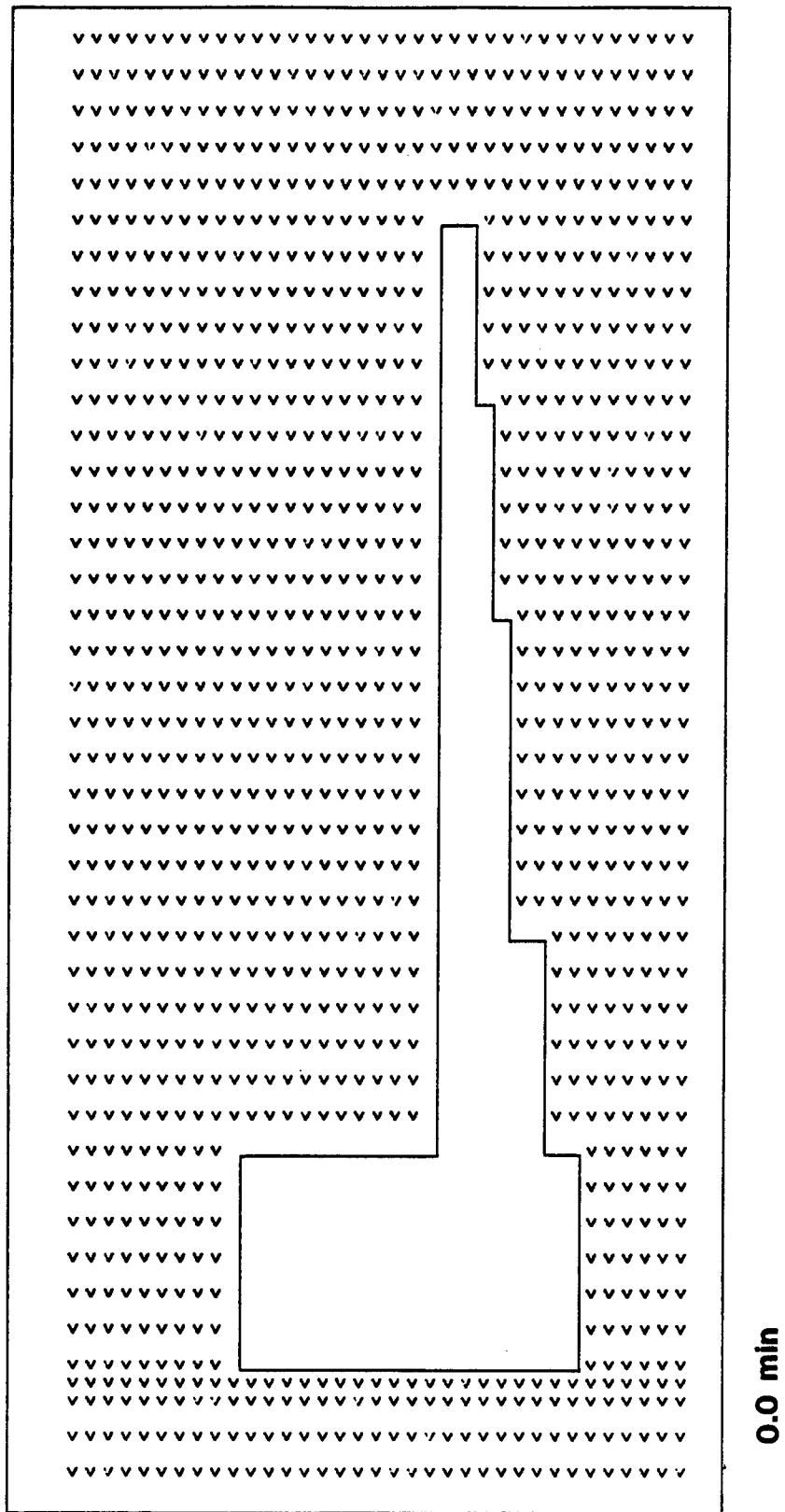
A significant objective of the evaluation routines is that, opposed to other existing analysis routines, the nodal mesh for finite element analysis is generated automatically with little interaction by the user. The discretization of the geometric model is the first of a series of steps that must be performed when solving a problem using FEM. Unfortunately, this particular step does not have a theoretical basis. It is an art and depends on the use of engineering judgment. The application of poor or improper judgment will produce inaccurate results even though all of the other steps are rigorously adhered to. The elements are chosen small enough to keep the computational effort within practical limits. In a three dimensional analysis of a complex casting several thousand nodes may be selected. The preparation of this input data, if done manually (i.e. by punching cards) would be very tedious and subject to human error. The logic used by the evaluation routines of the CAD/CAM Casting Process to generate a mesh is to select smaller elements close to the interfaces between sand and steel. It is at these interfaces where temperature gradients are the steepest. The computer routines automatically create an input file listing the coordinates of each node and the grouping of the nodes into elements.



**Figure III-3:** Drag half of stepped plate casting model used for mesh generation.

Another significant objective of the evaluation routines is to allow both two-dimensional and three-dimensional heat flow analyses. Two dimensional analyses of critical sections are economical in terms of computer time and analysis. Three-dimensional analysis will be useful for complex castings especially where lateral feeding is important.

The results of the heat flow analysis may be viewed in several forms. One graphical representation is illustrated by Figure III-4. The figure shows solidification progression maps computed at different times after pouring a plate casting similar in design to the ones used in the experimental portion of the program. Arrow marks over nodes indicate solid steel and sand, dash marks indicate steel in the mushy state (partially solid), and blank nodes indicate liquid steel.



**Figure III-4:** Simulated freezing progression map for stepped plate castings.

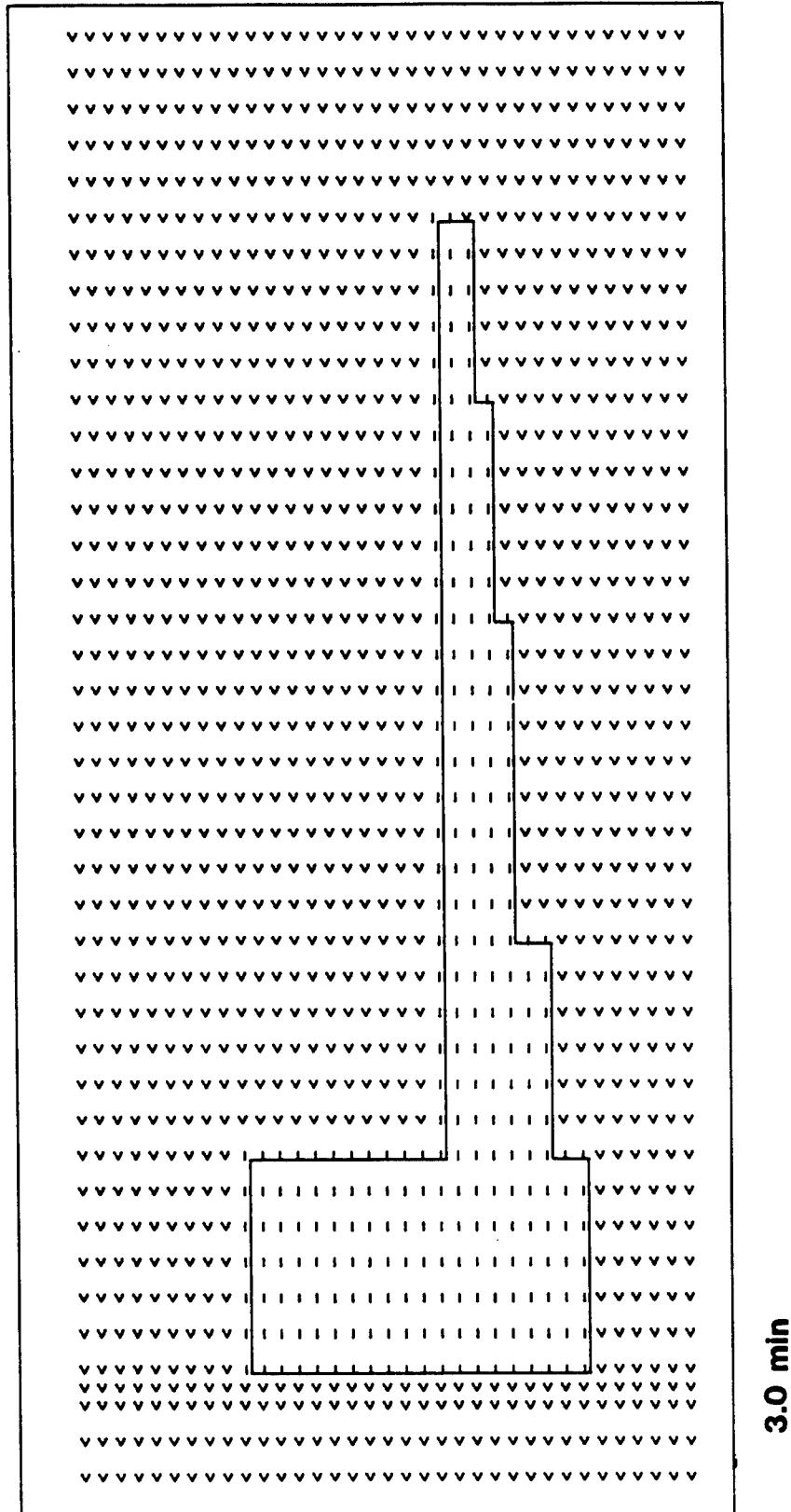


Figure III-4: Simulated freezing progression map, continued.

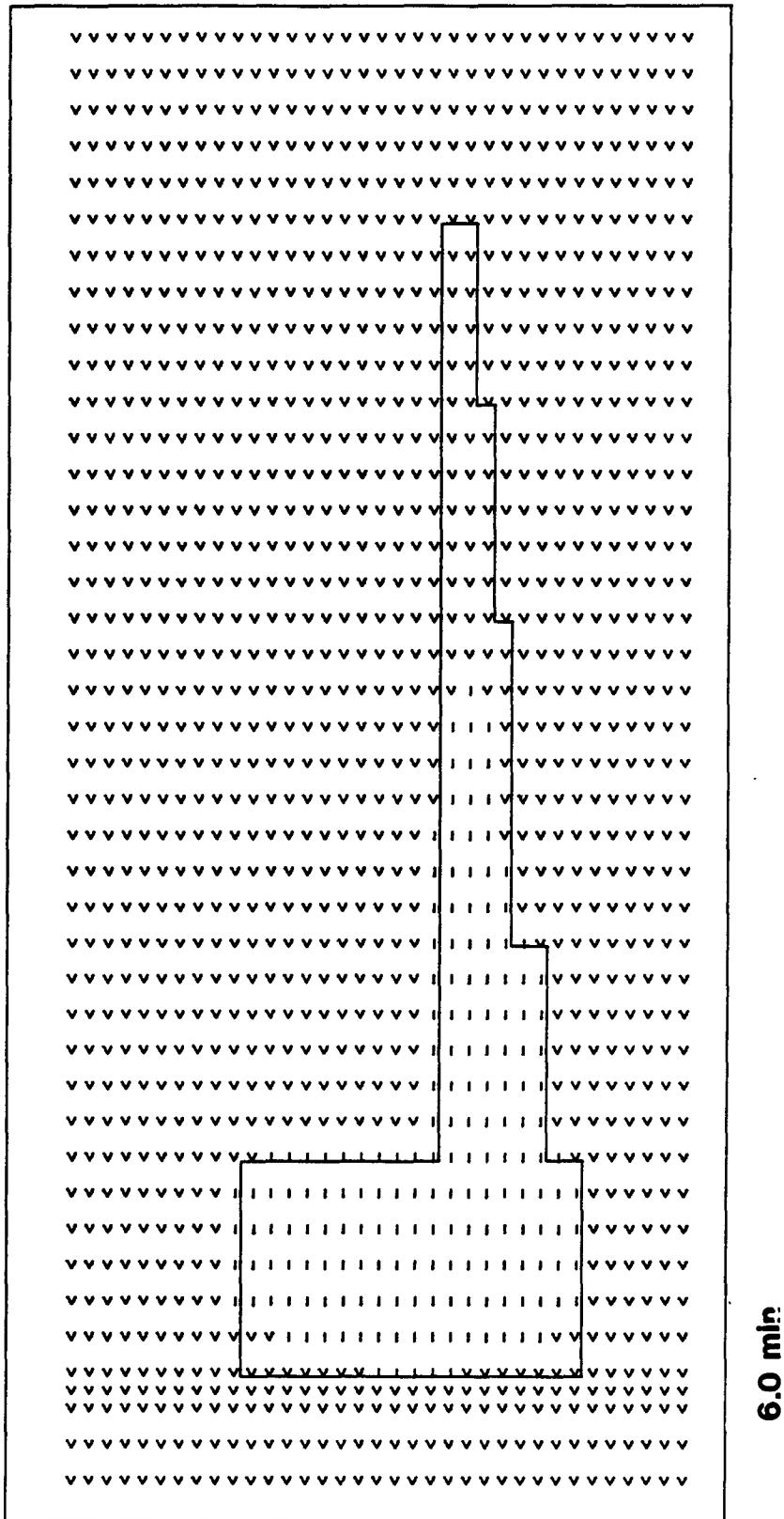
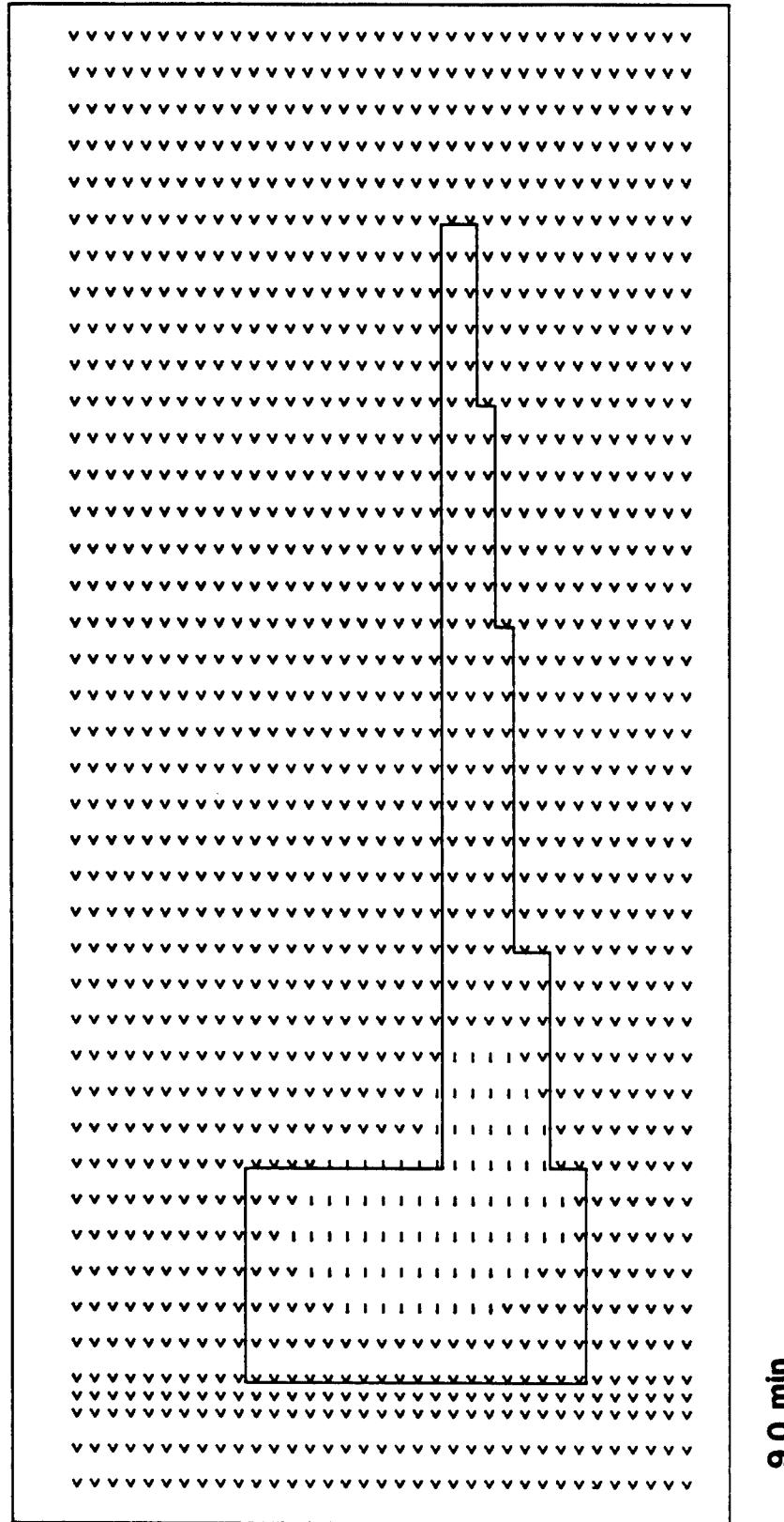
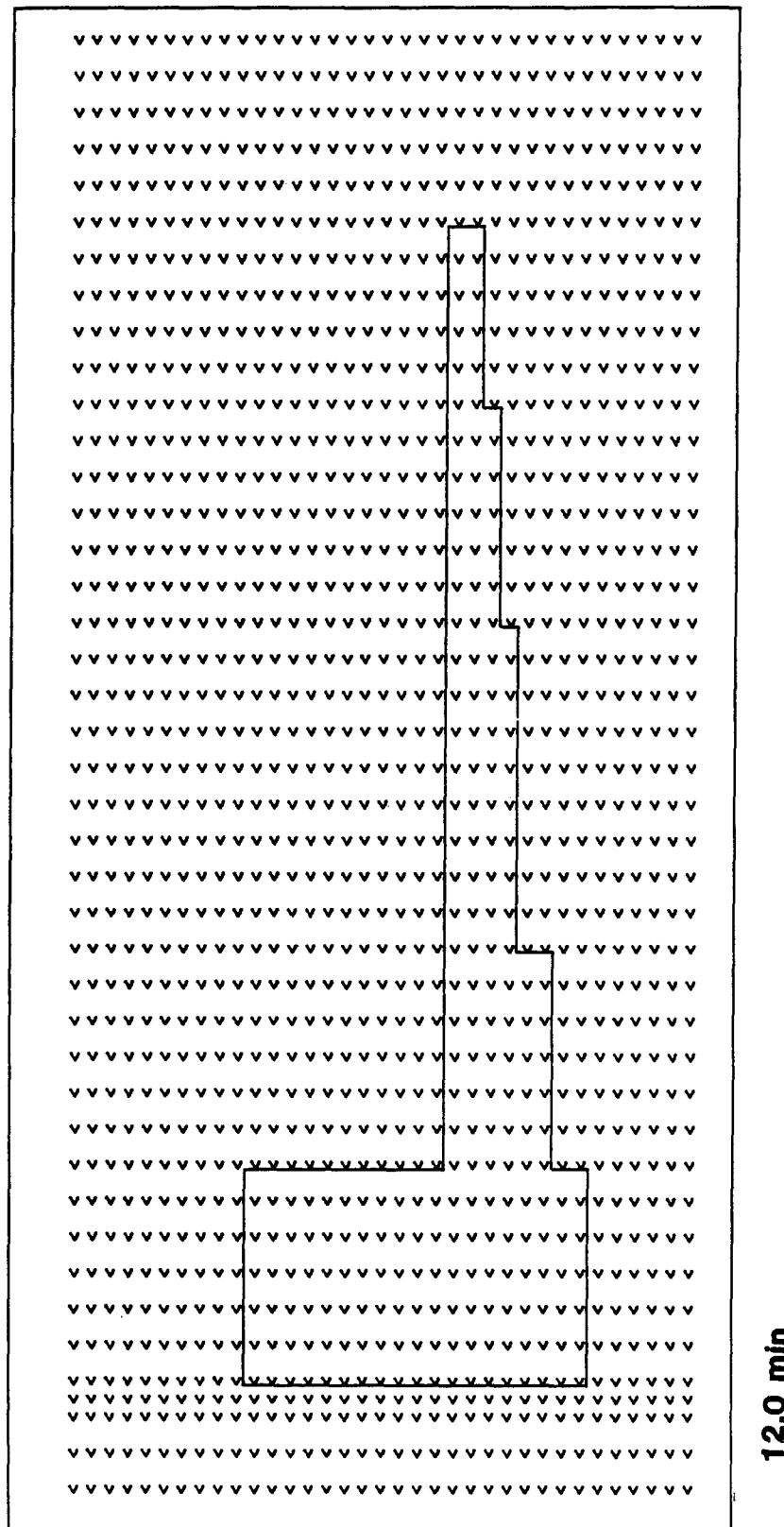


Figure III-4: Simulated freezing progression map, continued.



**Figure III-4:** Simulated freezing progression map, continued.



**Figure III-4:** Simulated freezing progression map, continued.

### III.4 EXPERIMENTAL WORK

One of the objectives of Phase I was to experimentally determine the thermal properties of the sand-alloy combination being simulated (cast armor steel - no bake sand) and to validate the computer simulations. To date eight different stepped plate castings were cast at Blaw Knox. The pattern is sketched in Figure III-5 and Figure III-6. The configurations of the plates are listed in Table III-1. The chemical analysis for the eight test plate castings are given in Table III-2.

The castings were instrumented for thermal analysis during freezing. Figure III-6 shows the thermocouple locations. The Pt/Pt-10%Rh thermocouple tips were located along the centerline. The thermocouple tips were protected by fused silica tubes. Millivolt readings for thermocouples 1-14 were recorded about every thirty seconds. The millivolt readings of thermocouples 15 and 16 were recorded continuously. The readings were entered, manually, into a computer for conversion to temperatures, plotting, and analysis. The castings were subsequently radiographed and heat treated.

The stepped plate casting configurations were simulated on the computer using the heat flow analysis programs. Comparisons are being made between the computer and the experimental analyses.

Figures III-7 through III-11 Show some of the analyses of the thermal measurements for two castings, A and D. Casting A has four steps and by conventional design rules this casting should be sound. Casting D has only two steps. The twenty inch long, two inch thick step would be expected by conventional design rules to show centerline shrinkage porosity, which was the case.

Figure III-7 shows cooling curves along the centerline for casting A. The numbers on the curves refer to the thermocouple locations given in Figure III-6. In the region above 2600 F (the freezing range) the curves are well separated indicating directional freezing toward the riser. Figure III-8 is another type of plot for casting A. In this case temperature is plotted versus distance. The numbers on the curves indicate the time after pouring. The slopes of these curves are temperature gradients. Again, the curves of Figure III-8 show the directionality of freezing for plate casting A.

Figure III-9 shows the cooling curves for casting D. Again the numbers on the curves refer to the thermocouple positions in Figure III-6. For this casting the cooling curves in the freezing range do not show directionality of freezing.

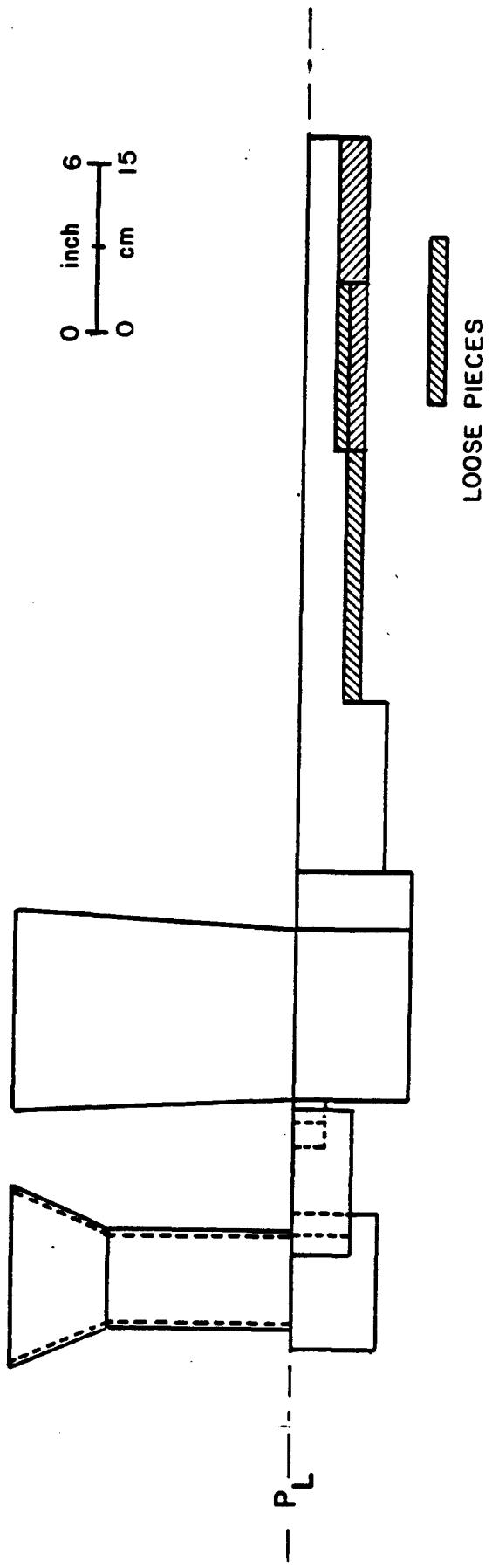


Figure III-5: Side view of pattern of test plate castings.

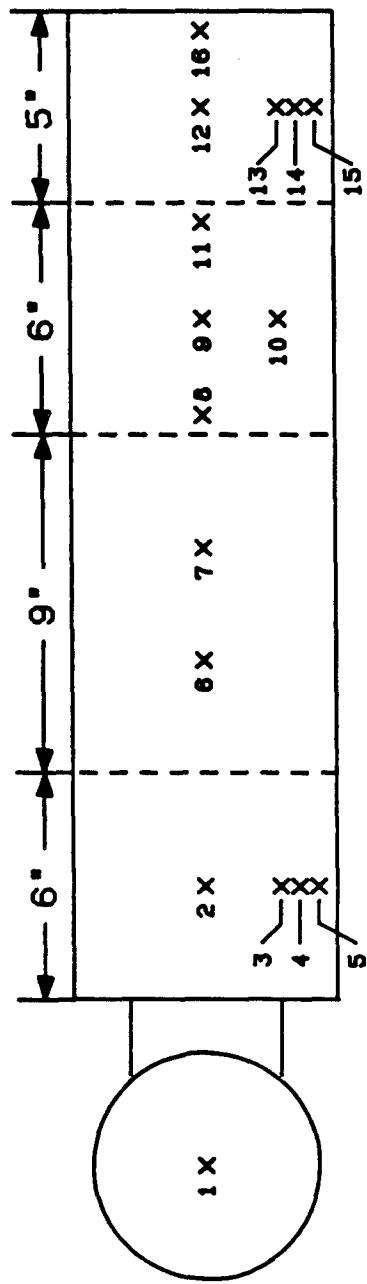


Figure III-6: Top view of test plate castings indicating thermocouple positions.

**Table III-1: Dimensions of Test Plate Castings.**

CASTING	SECTION 1	SECTION 2	SECTION 3	SECTION 4	CHILL
DESIGNATION	6-in. lg.	9-in. lg.	6-in. lg.	5-in. lg.	
A	3	2	1.5	1	no
B	3	2	1	1	no
C	3	2	2	1	no
D	3	2	2	2	no
E	3	1.5	1.5	1	no
F	3	1.5	1	1	no
G	3	2	2	2	yes
H	3	1.5	1	1	yes

**Note:** All castings were 7-inches wide.

Castings A-H were cast in no-bake sand.

Casting A, D, & G were cast in green sand.

Table III-2: Chemical Analyses (wt %) of Test Plate Castings.

<u>ELEMENT</u>	<u>Test Plate Casting Designation</u>							
	<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>	<u>F</u>	<u>G</u>	<u>H</u>
C	0.25	0.27	0.30	0.25	0.26	0.28	0.30	0.28
Mn	1.00	1.20	1.26	1.18	1.28	1.25	1.08	1.20
Si	0.42	0.42	0.40	0.46	0.40	0.34	0.42	0.45
S	0.026	0.026	0.027	0.028	0.026	0.028	0.026	0.027
P	0.027	0.025	0.029	0.027	0.025	0.027	0.028	0.025
Cr	0.98	1.05	1.16	1.04	0.99	1.12	1.01	1.15
Ni	1.01	1.10	1.16	1.19	1.02	1.10	1.10	1.09
Mo	0.48	0.46	0.50	0.48	0.45	0.49	0.51	0.46

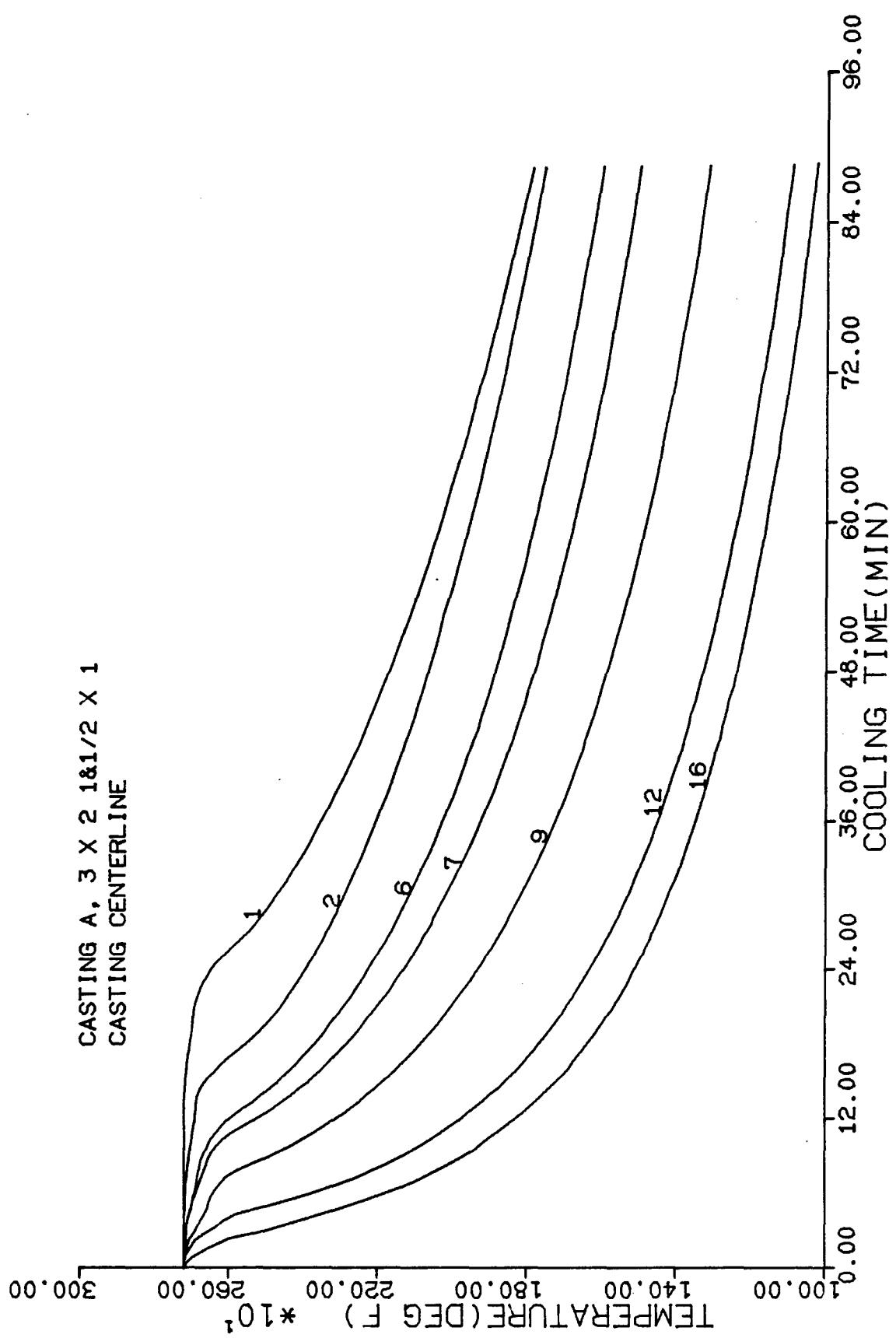
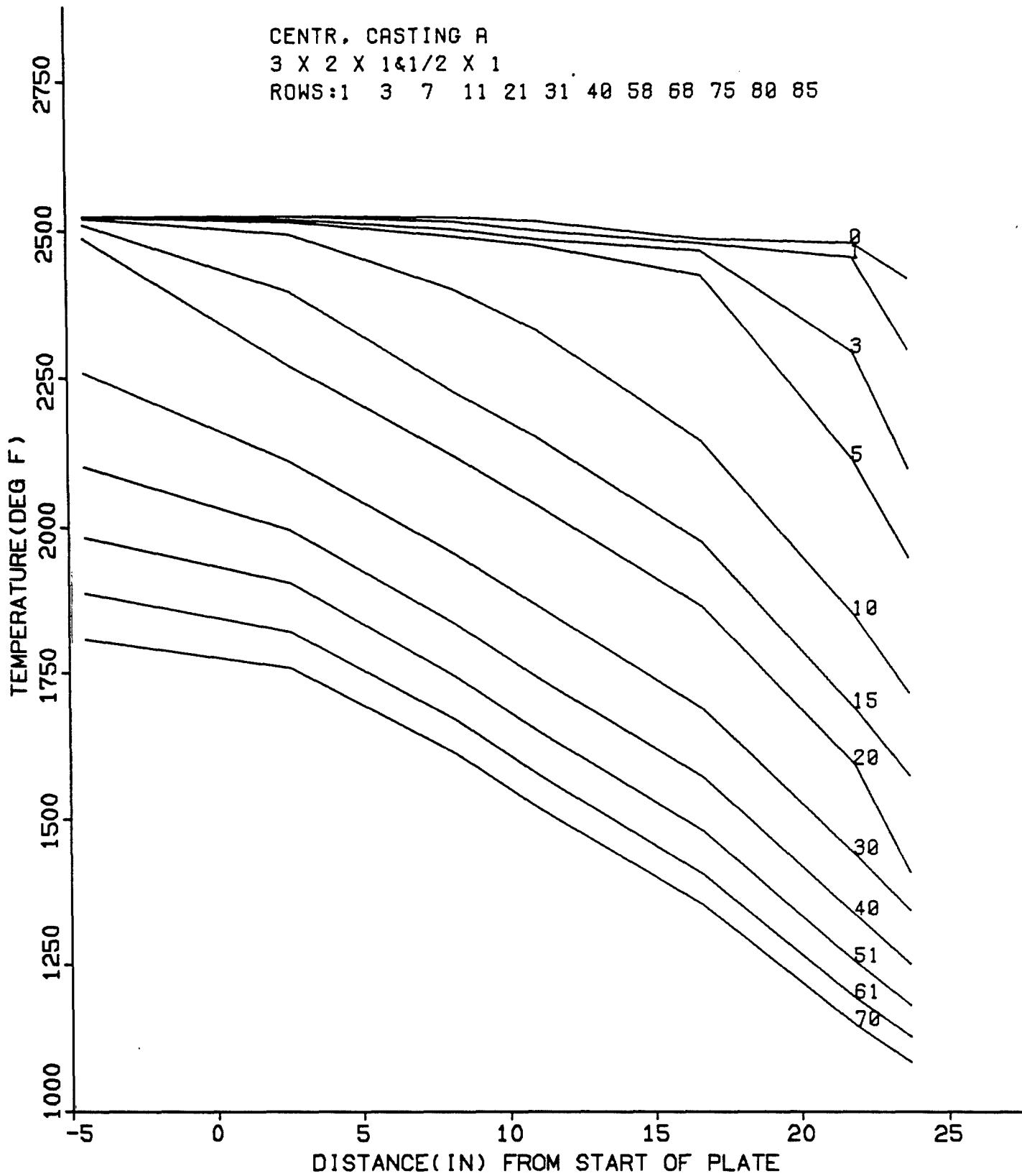


Figure III-7: Cooling curves for casting A at centerline positions.



**Figure III-8:** Temperature distribution along centerline of A at several times.

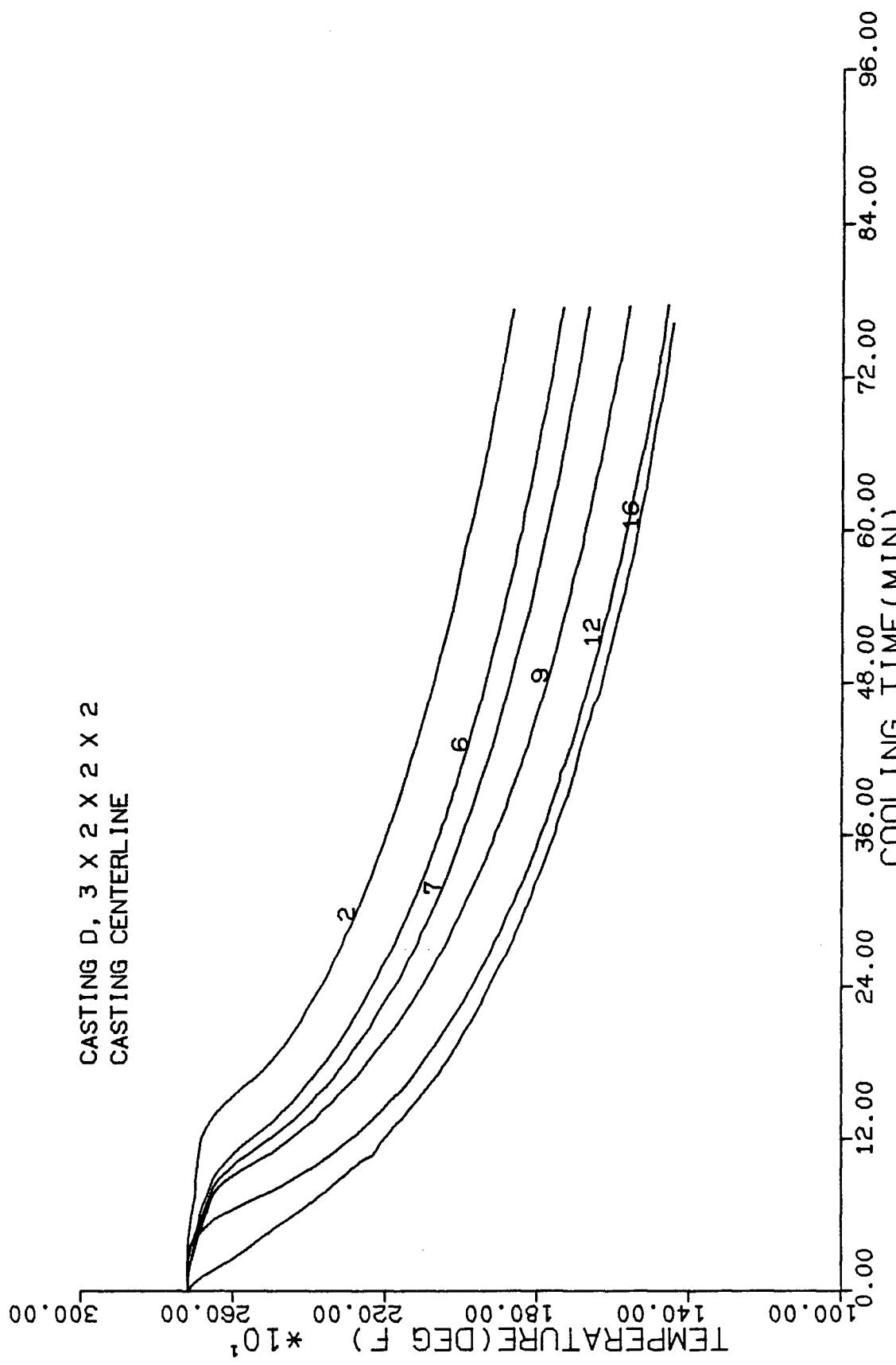


Figure III-9: Cooling curves for casting D at centerline positions.

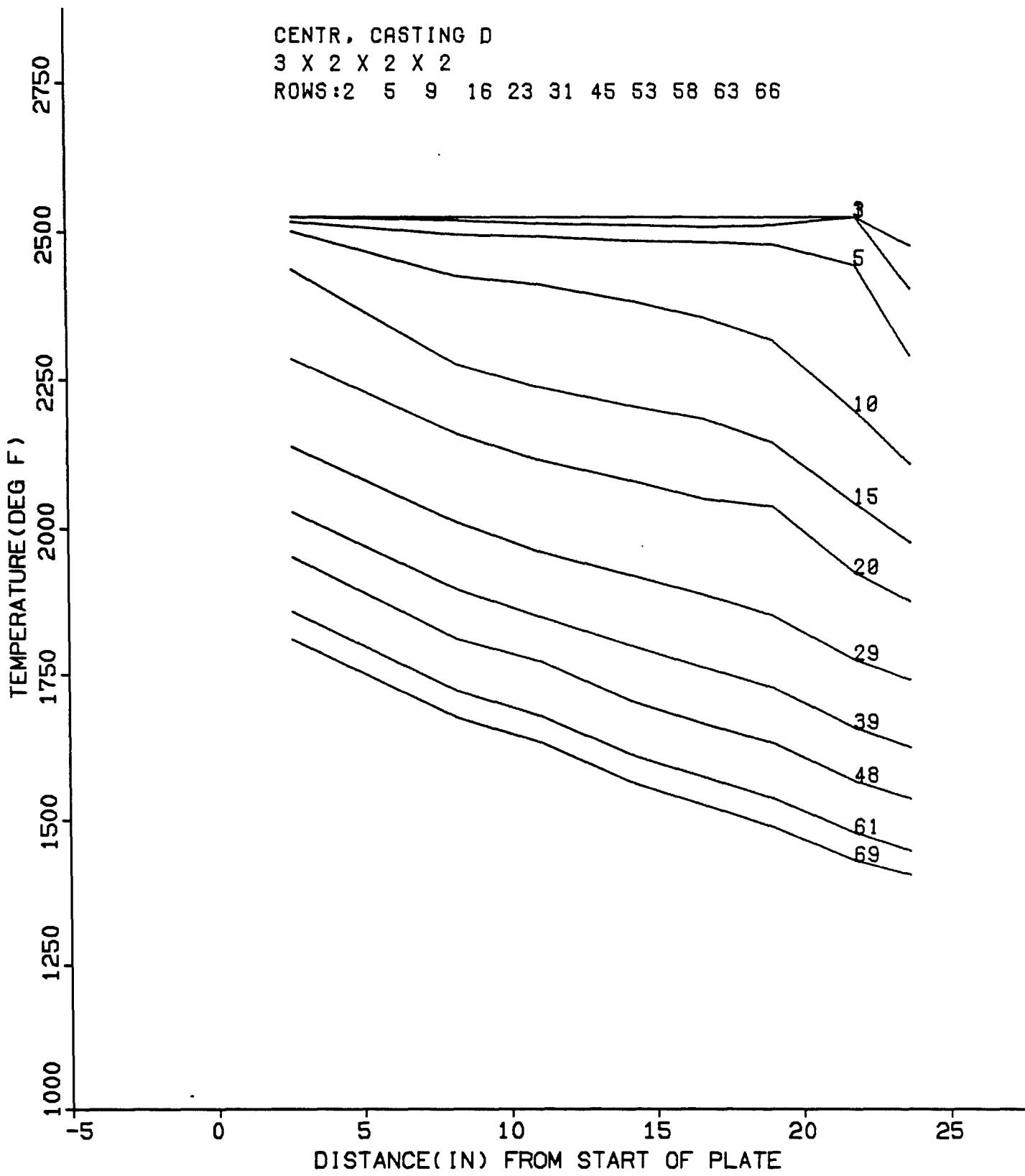


Figure III-10: Temperature distribution along centerline of D at several times.

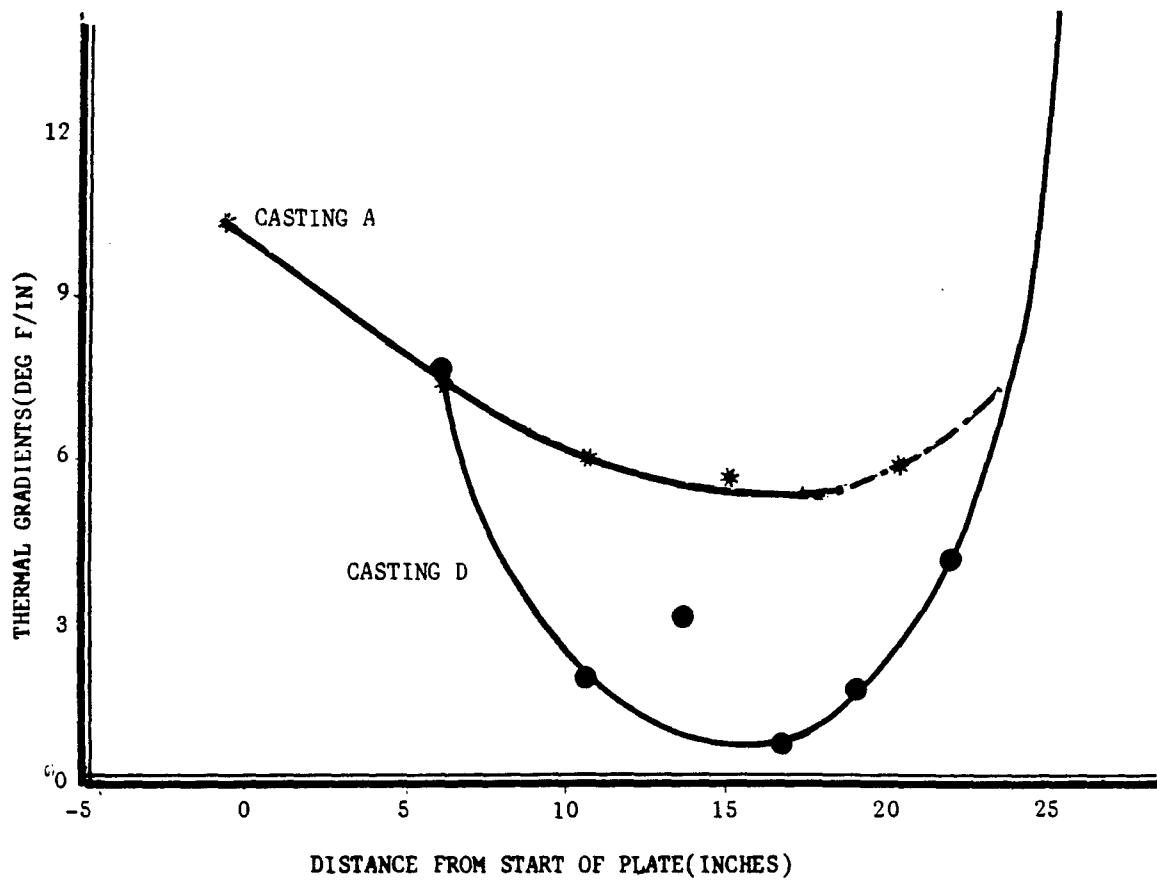


Figure III-11: Thermal gradients along centerline in freezing range for A and D.

Comparison of Figures III-8 and III-10 indicate, as expected, shallower thermal gradients in casting D.

Figure III-11 indicates the thermal gradients  $^{\circ}\text{F/in.}$  measured within the freezing range along the lengths of the two castings. Casting A shows gradients over  $3^{\circ}\text{F/in.}$  at any point. Casting D, on the other hand, shows very low gradients in the center of the two-inch plate. The gradient increases near the riser and near the end of the plate (end effect).

The solidification progression maps in Figure III-4 are computed for a casting with a stepped plate configuration the same as casting A (however the riser in the computer simulated casting was smaller). The computed maps show a pattern of directional freezing. The computed analysis has also been used to show graphically the cooling curves, the temperature versus distance plots, and the thermal gradients in the freezing range plotted versus location along the centerline.

### **III.5 FURTHER WORK**

In the subsequent development of the CAD/CAM Casting Process, the experimental thermal data obtained from the test plate castings will be incorporated into the programs.

Mechanical properties and radiographic soundness of the test plate castings will be correlated with the computed and experimentally determined thermal conditions during freezing of the plate castings. These correlations will be used to generate criteria amenable to computer application for predicting casting soundness.

A number of armor castings of increasing complexity will be simulated, cast and experimentally analyzed.

The techniques of the CAD/CAM Casting Process will be tested on various castings in order to develop their potential. Computer routines developed at the University of Pittsburgh for analysis of gating systems will be incorporated into the CAD/CAM Casting Process. These routines compute filling times and heat loss in a given gating system.

### **III.6 SUMMARY**

The CAD/CAM Casting Process represents an important step in the application of computers to casting engineering. Its ultimate goal is to provide working techniques for the design of better castings in a fraction of the time presently required.

A system of programs incorporates regular computer aided drafting techniques plus a number of routines relative to heading and gating.

Automatic mesh generation tied together with 2-D and 3-D finite element analysis allows for expedient heat flow simulations of castings in sand molds. It makes possible early casting soundness predictions in so far as they are related to design.

Experimental heat flow simulations of eight test castings were used to determine heat transfer parameters for the alloy-sand combination. Computer simulations for the same test castings were found substantially in agreement with the experimental results.

#### **ACKNOWLEDGEMENTS**

The CAD/CAM Casting Process is being developed under the sponsorship of the U.S. Army, Tank and Automotive Research and Development Command (Contract No. DAAK30--78-C-107). Mr. Thomas Wessel is the Technical Officer for the program. Messrs. John Gasper and John Farinelli and Ms. Jeri Frizza assisted in the thermal analyses. Messrs. John Duggan, Edward Ruhe, and James Echlin extended advice and assistance in the project.

## IV. SIMULATION OF HEAT FLOW IN CASTINGS

H. D. Brody and R. A. Stoehr  
University of Pittsburgh  
Pittsburgh, Pennsylvania 15261

### SUMMARY

The use of computers to simulate casting, ingotmaking and welding processes and the inherent solidification phenomena has expanded many-fold in the past few years. Indications point to even more extensive use of computer models as research tools as well as adjuncts to process control and process design. Using some heat flow models as examples, this paper illustrates the blending of numerical and computer techniques to simulate casting processes.

\* Originally published: H. D. Brody and R. A. Stoehr, "Computer Simulation of Heat Flow in Casting," Journal of Metals, TMS-AIME, Sept 1980, pp. 20 - 27.

## IV.1 INTRODUCTION

Simulation techniques have been applied to casting processes over an extended period. The simulations have been aimed at increasing the understanding of solidification phenomena and at obtaining relevant operating and materials parameters. Recently, the application of computer techniques to the simulation of manufacturing processes, including casting, has expanded widely, and more comprehensive and complex models of the processes and the inherent solidification phenomena have been developed. Substantial interest has been expressed in developing computer models to where they can be useful for process control and for the design of manufacturing processes. The growing interest in computer simulation of solidification was evidenced at the recent conference on Modeling of Casting and Welding Processes sponsored by AIME, ASM, and the Engineering Foundation.<sup>1</sup>

Microsegregation (or coring) is a casting phenomena that has received consideration over an extended period. In 1913, Gulliver published a model of the development of microsegregation using algebraic techniques.<sup>2</sup> In 1942, Scheil, using calculus, developed a simulation of coring based on the same simple assumptions as Gulliver.<sup>3</sup> Beginning in the mid-60s and continuing actively to the present, numerical methods and computer techniques have been blended to develop models that simulate the development of microsegregation, making less restrictive assumptions concerning transport properties of the alloy, complexity of the phase relations, and complexity of the solidification morphology.<sup>4-8</sup> The microsegregation models are not used directly in process control. However, the concepts and portions of the models have been incorporated in simulation of other casting phenomena, e.g. heat flow, macrosegregation, inclusion formation, and thermal stress,<sup>9-14</sup> which may be incorporated in process control models.

Solidification processes cover a wide range of scale, economics, and microstructural control, e.g. welding, sand and investment casting, ingot-making, electroslag and vacuum arc remelting, crystal growth, powder atomization, laser glazing, etc. Models have been developed for heat flow, bulk fluid flow, mass transport on the microstructural and atomic scales, interface kinetics, and thermal stress and deformation.<sup>1</sup> The more macroscopic aspects of the casting processes, heat transfer, thermal stress, and fluid flow are receiving considerable attention and are the most likely aspects to be incorporated in process control and process design models, as is already happening.<sup>1</sup> However, the use of models simulating microscopic phenomena controlling microstructural features of cast alloys can be developed to allow the design of casting processes to control and optimize casting, ingot, and weld soundness and the properties that depend on microstructural features.<sup>14</sup>

Key steps in developing a model are the idealization of the system incorporating reasonable assumptions about the process, collection of reliable input values for casting and alloy parameters, and validation of the model. Developing a computer model, whether aimed at increasing understanding or at process control, is an iterative, two-track process involving sequences of experimentation and of model improvements. In research, the model or experiments may come sequentially or concurrently. The model will be based on assumptions and data derived from and tested by experiment, the experiments given direction by the predictions and the requirements of the model. At each round the models will be based on an expanded data base and increased understanding, and the experiments will be refined based on the directions indicated by the model and the capability to test theories on a quantitative basis.

Validating the model is critical. As the models are applied to complex manufacturing processes, they will commonly be applied beyond the range of verification. For these extrapolations to be realistic, the models must be built on a reliable data base obtained by carefully checking the measurable aspects of the process. Indeed, the requirement to find more information on the casting process and the alloy in order to build a data base for and/or validate a model will result in information directly useful to improving the process or its operation independently of the computer model.

Incorporating computer simulation in process control or design will often enforce a discipline of operation that leads to more consistent application of the process. And, as the model is continually compared against results, an expanded data base will be developed, leading to improved representation by the model.

Finite difference<sup>15</sup> and finite element<sup>16</sup> models are approximate numerical techniques that are frequently used to simulate casting processes. The finite difference method (FDM) was used in conjunction with table-top calculators (and as a graphical technique), but its power and use have been expanded by application on modern computers. The finite element method (FEM), requiring manipulation of large matrices, is practical only in conjunction with a high-speed, large-memory computer system. A facility for representing the solution (output) graphically is desirable for both numerical methods.

This paper illustrates the finite difference and finite element techniques. Heat flow examples in pulsed welding, the casting of large steel and iron rolls, D.C. continuous casting, and complex shape sand casting have been selected for the illustrations. The techniques can simulate other casting processes and other aspects of the processes, as listed above, i.e. fluid flow, thermal stress, etc.

## IV.2 GENERAL HEAT CONDUCTION EQUATIONS

The same expressions are the basis for numerical and analytical methods. The governing differential equation for conduction within a localized region of a casting may be written

$$\rho C_p \frac{\partial T}{\partial t} = \nabla(\kappa \nabla T) + \rho C_p R \cdot \nabla T + Q \quad (\text{IV.1})$$

where the term on the left represents the heat accumulation (or loss) within a differential region of interest; the first term on the right is the difference between the heat conducted into and out of the differential element; the second term represents heat transported as a result of bulk material flow (e.g. as in continuous casting or crystal growth); and the final term is the heat input (or loss) from a heat source (or sink). For a one-dimensional model with no bulk movement and constant thermal properties Equation (IV.1) would reduce to

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + Q \quad (\text{IV.2})$$

where

$$\alpha = \kappa / \rho C_p$$

Solving these equations to yield temperature as a function of time and position requires stipulating an initial condition and stipulating conditions at the boundaries.

Using the techniques of calculus, these equations have been solved for a variety of geometric configurations and boundary conditions.<sup>17,18</sup> However, for simulation of casting processes that include some of the normal complexities of geometry, variation of thermal properties with temperature, and particularly the localized evolution of the latent heat of fusion, solutions based on calculus are not readily available. Numerical techniques, executed by computer, are a practical and sometimes the only route to a solution.

### IV.3 FDM FOR PULSED WELDING

Consider heat flow in pulsed arc welding of thin sheets as an illustration of the finite difference technique. Refer to Figures IV-1, IV-2, and IV-3. In the pulsed arc welding process, the power is cycled from a peak rate of heat input  $q_p = i_p e_p / 4.18$  (cal/s) over a high pulse time  $t_p$  to a reduced level  $q_b = i_b e_b / 4.18$  over a background pulse time  $t_b$ . The product of the period and the welding speed  $v \cdot (t_p + t_b)$ , the distance the arc moves in one pulsing cycle, is called the pitch. Pulsed arc welds with long pitch and small  $t_p/t_b$  ratios would appear as a series of discontinuous spot welds. Short pitch, high  $t_p/t_b$  pulsed arc welds would appear as continuous welds. Typically in pulsed welding, the size of the weld pool increases and decreases in size and the temperature gradients ahead of, normal to, and behind the weld pool change with time within a pulsing period.

For numerical simulation, the region of concern—the sheet around the weld pool—is divided into a grid of elements, square cross section blocks with the same thickness as the sheet; and each element is characterized by a node at its center. Because of symmetry, only one-half the sheet need be considered, and the elements are made smaller (and the nodes are closer together) near the weld pool to get better resolution in the region of interest from a solidification standpoint. A mesh of small spacing,  $\Delta X_s$ , moves with the arc and includes the weld zone. A node at the center of each element is assumed to have the average temperature of the element. The rows and columns of the array of nodes will be numbered for reference:

small grid rows	$1 < J < N_s'$
small grid columns	$1 < I < M_s'$
large grid rows	$1 < K < N,$
large grid columns	$1 < L < M.$

Thus,  $T_{K,L}$  refers to the temperature used to represent the element and node in the  $K^{\text{th}}$  row and  $L^{\text{th}}$  column of the large grid.

In the finite difference procedure, total and partial derivatives such as in Equation (IV.1) and (IV.2) are represented by differences, for example:

$$\frac{\partial T}{\partial t} \approx \frac{T_{K,L} - T_{K,L}}{\Delta t}$$

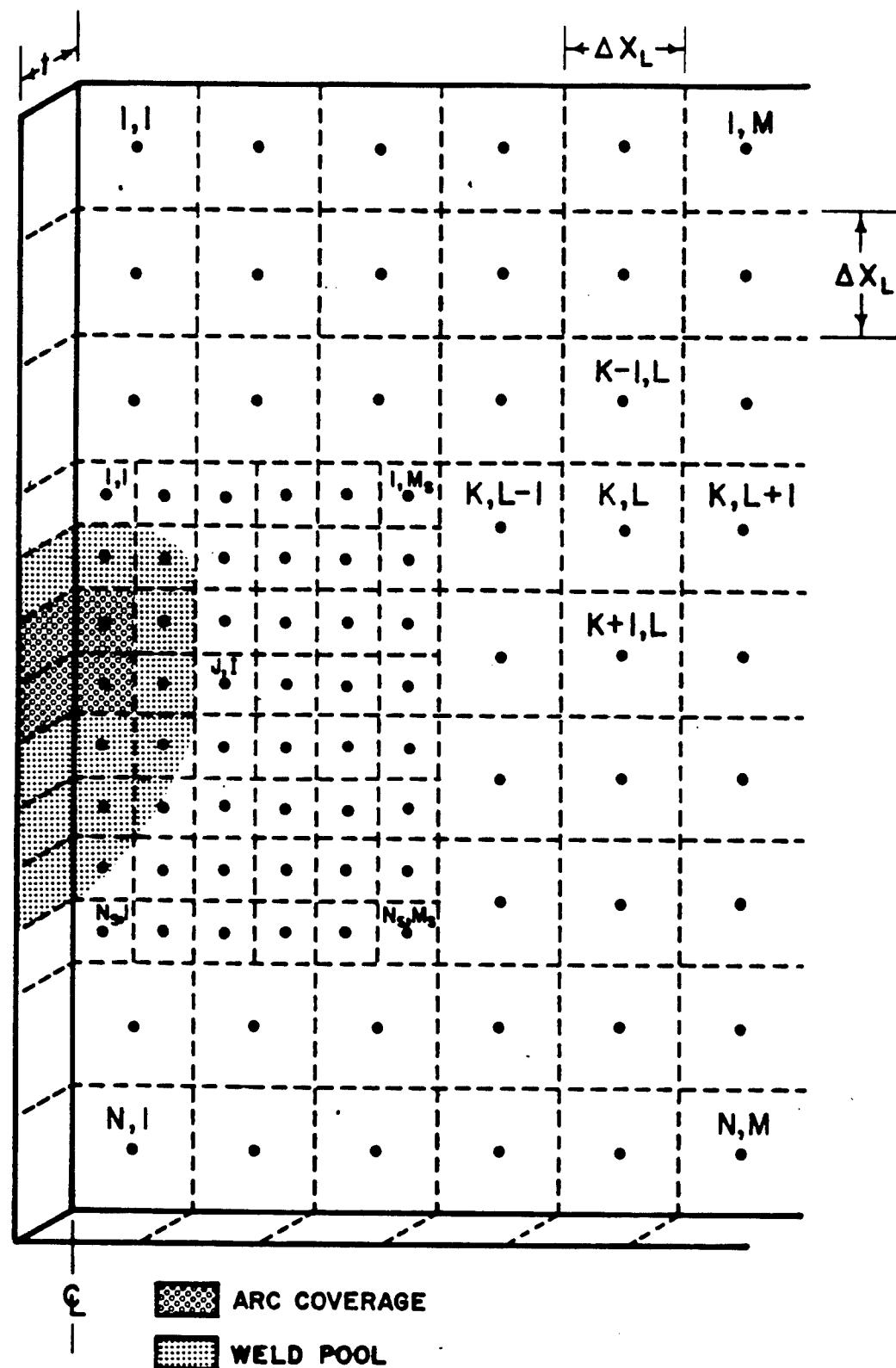
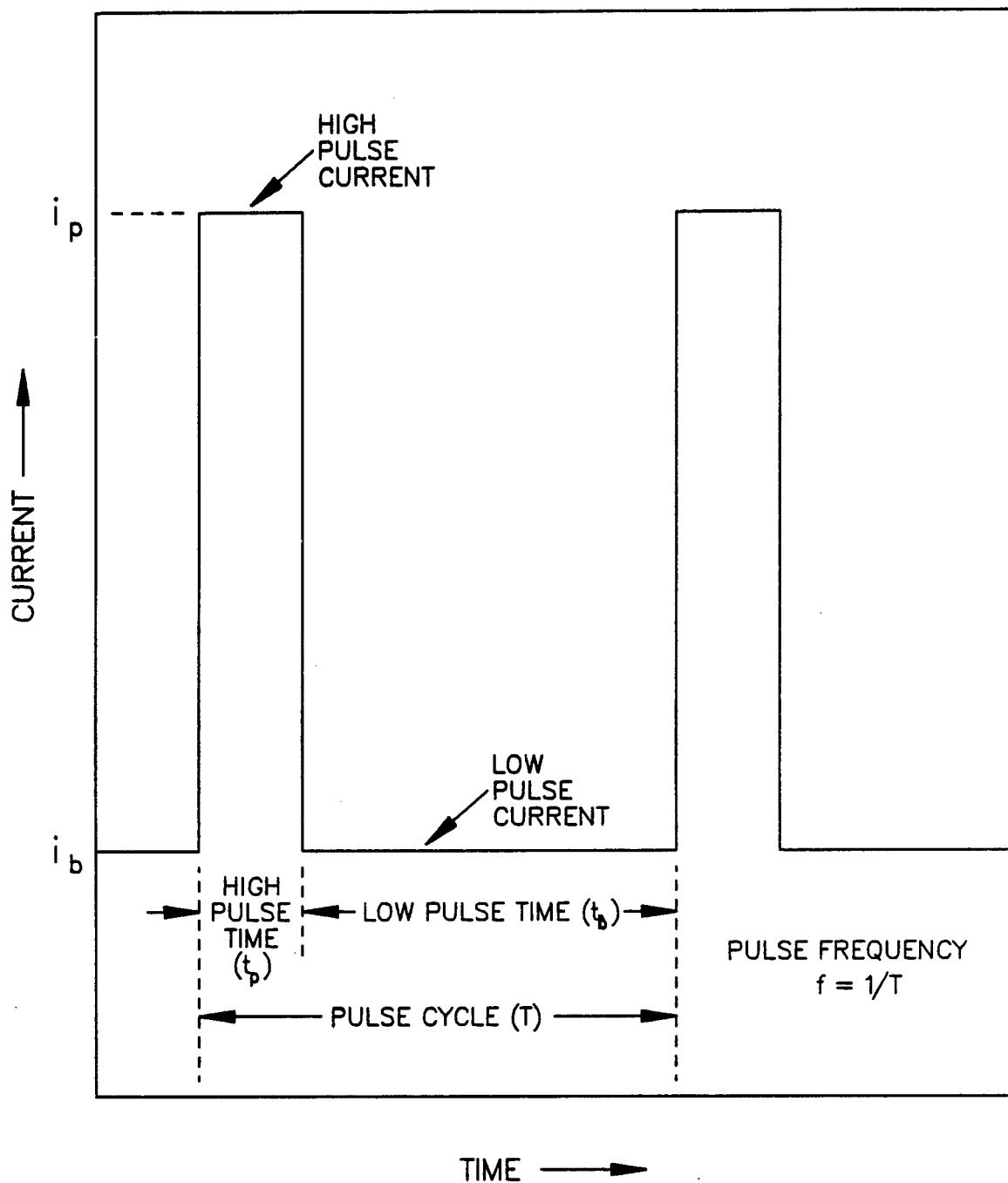


Figure IV-1: Example of element mesh suitable for pulsed arc welding.



**Figure IV-2:** Definition of terms used in pulsed arc welding.

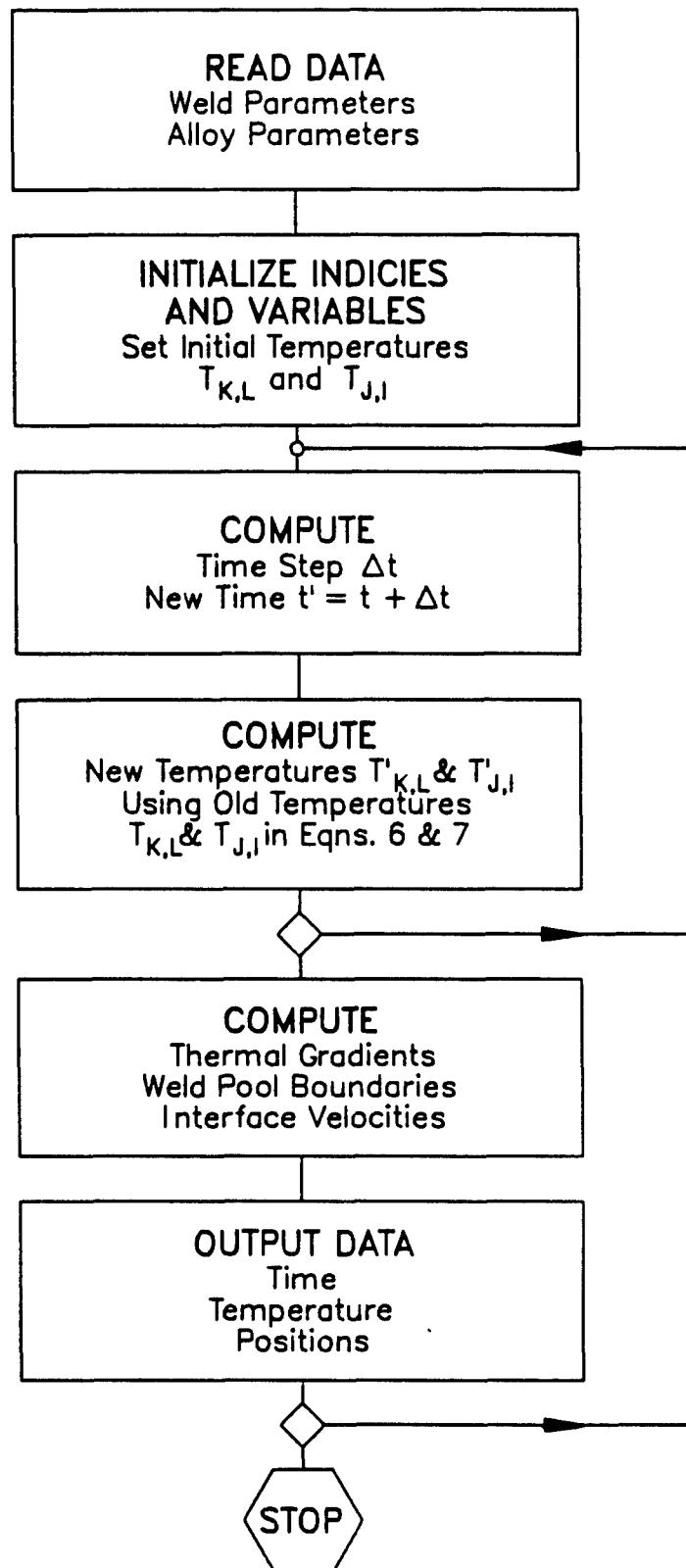


Figure IV-3: Flow chart for finite difference analysis of pulsed GTAW.

$$\frac{\partial T}{\partial X} \approx \frac{T_{K,L} - T_{K-1,L}}{\Delta X_L} \quad (IV.4)$$

$$\frac{\partial^2 T}{\partial X^2} \approx \left\{ \frac{T_{K+1,L} - T_{K,L}}{\Delta X_L} - \frac{T_{K,L} - T_{K-1,L}}{\Delta X_L} \right\} \left\{ \frac{1}{\Delta X_L} \right\}$$

First, to illustrate the method only, Eqn. (IV.2) for one-dimensional heat flow is rewritten below in finite difference form:

$$\rho C_p \frac{T_K - T_{K-1}}{\Delta t} = \frac{\kappa}{\Delta X^2} \left\{ \frac{T_{K+1} - T_K}{\Delta X} - \frac{T_K - T_{K-1}}{\Delta X} \right\} + S \quad (IV.5)$$

which simplifies to

$$T_K = T_{K-1} + M \{ T_{K+1} + T_{K-1} - 2 T_K \} \quad (IV.6)$$

where

$$M = \alpha \Delta t / \Delta X^2$$

Note only one subscript is used in this illustration for one-dimensional heat flow.

In considering pulsed arc welding of sheets, two-dimensional heat flow is computed and two subscripts are necessary to define the nodes. Heat is assumed to be conducted into (or out of) each element from its four nearest, orthogonal neighbors. For two-dimensional heat flow, the finite difference algorithm for conduction heat flow within an internal element in the large grid region would be written:

$$T_{K,L} = T_{K,L} + M \{ T_{K+1,L} + T_{K-1,L} + T_{K,L+1} + T_{K,L-1} - 4 T_{K,L} \} \quad (IV.7)$$

The heat from the arc source may be introduced (with some weighting factors) for those elements on the centerline covered by the arc

$$T_{J,1} = T_{J,1} + M_s \{ T_{J+1,1} + T_{J-1,1} + T_{J,2} - 3 T_{J,1} + (q/\kappa t) \} \quad (IV.8)$$

The relative motion of the arc and the sheet can be taken into account by adding a term such as the second term on the right in Eqn. (IV.1) or by changing (by the computer program) the location of the node receiving external heat input, using the algorithm (IV.8), with the proper timing.

The initial condition is imposed by adjusting the first values of  $T_{K,L}$  and  $T_{K,L}$ ; for example, setting all initial temperatures to  $T_0$ . A value of the time step  $\Delta t$  is selected so that

$$M = \{\alpha \Delta t / \Delta X^2\} < 1/4 \quad (IV.9)$$

The value of  $\alpha$  may be a function of temperature (and position or composition), and the largest value of  $\alpha$  should be used in determining condition (IV.9). A time step selected to satisfy condition (IV.9) for the small grid would be suitable for the large grid. It may be desirable, to save computer time, to use different time steps for the large and small grids. Small values of  $M$  will give greater accuracy. Values of  $M$  approaching 1/4 will conserve computer time. Values of  $M$  greater than 1/4 may give instabilities in the solution.

The boundary conditions are enforced on the solution by the way the elements on the boundaries are treated. For example, due to symmetry at the centerline, the left boundary may be treated as adiabatic. Note in algorithm 6b that no heat is assumed to enter from the face on the centerline. Note also in algorithm 6a that the top and bottom surfaces of the sheets are considered adiabatic. Heat loss by convection and radiation could be added.

A simplified flow diagram for the computer program to use this explicit finite difference scheme for simulating pulsed arc welding is shown in Figure IV-3. Details on the actual computation scheme are given elsewhere<sup>1,19-21</sup>.

To obtain reasonable input data for the model and to validate the model, two series of experimental welds are being made on Fe-26% Ni alloy and 321 and 316 stainless. In addition to measuring weld zone dimensions on all welds, thermocouples are being placed in selected sheets during welding and high-speed motion pictures are being taken of the weld zones<sup>1,19-21</sup>. Examples of the computer predictions are given in Figures IV-4 and IV-5. Detailed comparisons of the experimental values and the computer predictions can be found in the appropriate references<sup>1,21</sup>. Computer simulations of the welding process are suitable for understanding and controlling weld structure, weld cracking, and weld distortion<sup>1</sup>.

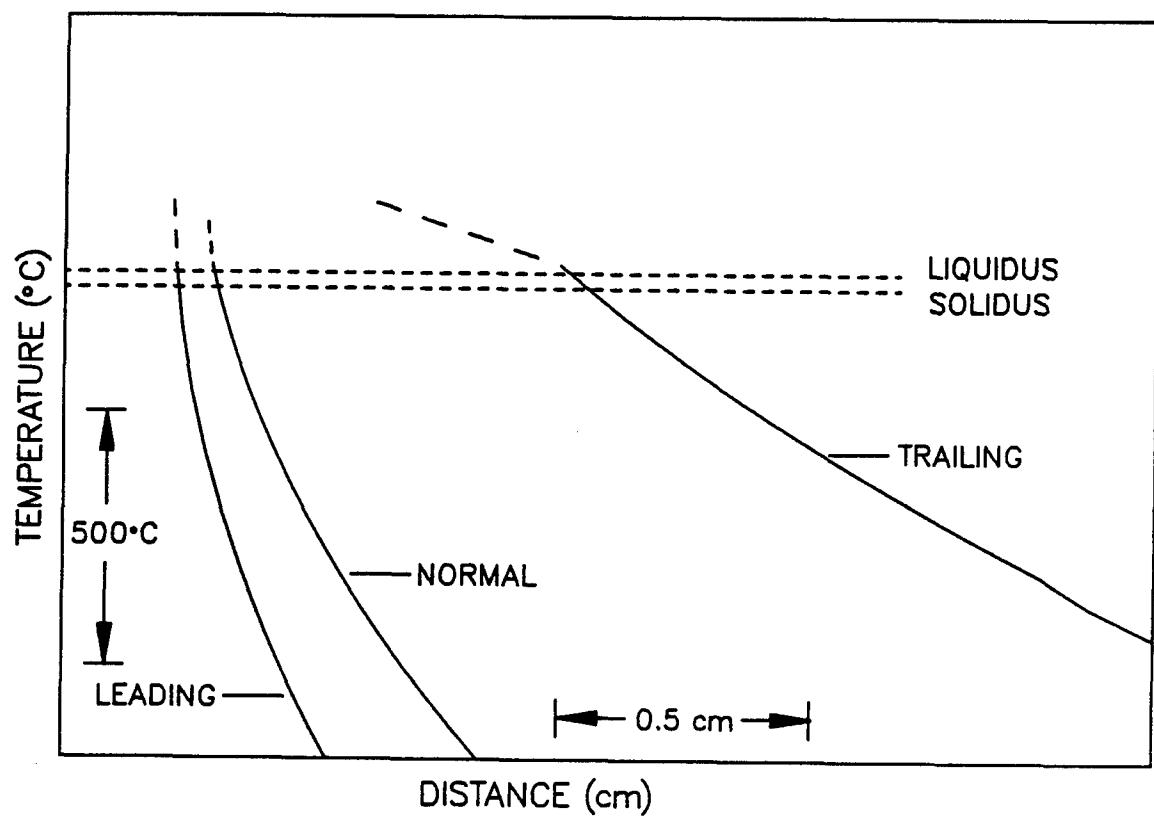


Figure IV-4: Thermal profile at change from peak to background power.

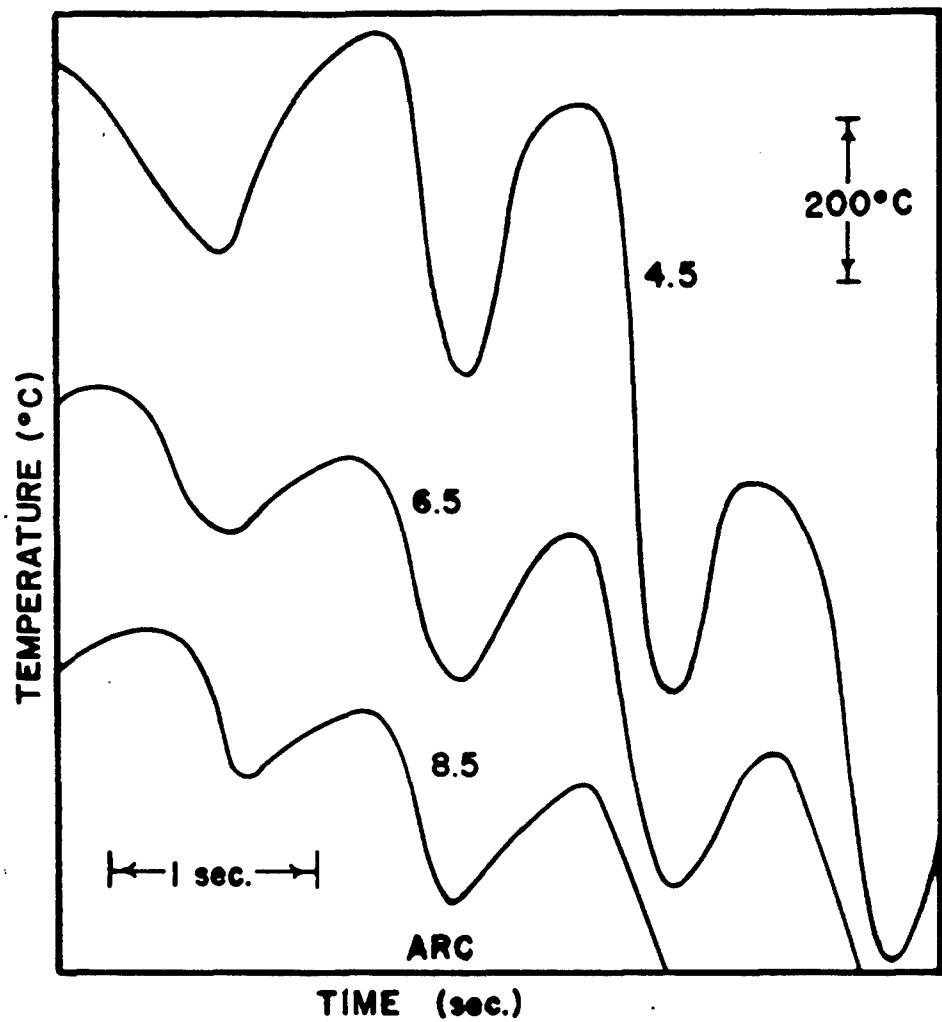


Figure IV-5: Computed temperature variation in pulsed arc welding.

## IV.4 FDM FOR ROLL CASTINGS

Considering the cost of experimentation when casting 10,000-500,000 lb parts makes clear the potential value of computer simulations of the solidification of large steel and iron rolls. One requirement, to avoid gross internal shrinkage porosity, could be aided by computer simulation of heat flow in large complex molds that predict the motion of the liquidus and solidus isotherms. Figure IV-6 shows a hypothetical design for a mold for a large steel roll casting. The heat flow within this mold has been simulated by an implicit difference computer simulation<sup>22</sup>.

The implicit finite difference technique differs from the explicit finite difference technique, outlined above, in that the new temperature (temperatures at the end of the timestep  $\Delta t$ ) and the thermal properties at the end of the timestep are used to calculate the set of new temperatures. For illustration, Eqn. (IV.6) for one-dimensional conduction is rewritten in implicit finite difference form:

$$T_K = T_K + M' \{T_{K+1}' + T_{K-1}' - 2 T_K\} + Q \quad (\text{IV.10})$$

which can be rewritten for  $Q = 0$ :

$$T_K = - M' T_{K-1}' + (2 M' + 1) T_K - M' T_{K+1}' \quad (\text{IV.11})$$

In the implicit finite difference method, an expression of the form of Eqn. (IV.11) can be written for each element. Each equation contains several unknowns. Altogether, there are an equal number of unknowns and equations to be solved simultaneously. Using matrix notation the equations to be solved may be written:

$$\{T\} = [M'] \{T'\} \quad (\text{IV.12})$$

and the solution is represented by

$$\{T\} = [M']^{-1} \{T'\} \quad (\text{IV.13})$$

where

$\{T\}$  is the column matrix of new temperatures,

$[M']$  is the square matrix of coefficients (most of which will be zero and not stored in the computer memory),

$\{T\}$  is the column matrix of old temperatures.

Rather than matrix inversion, the simultaneous equations are generally solved by elimination or iteration<sup>23</sup>.

The solidification of large roll castings has been simulated using a two-

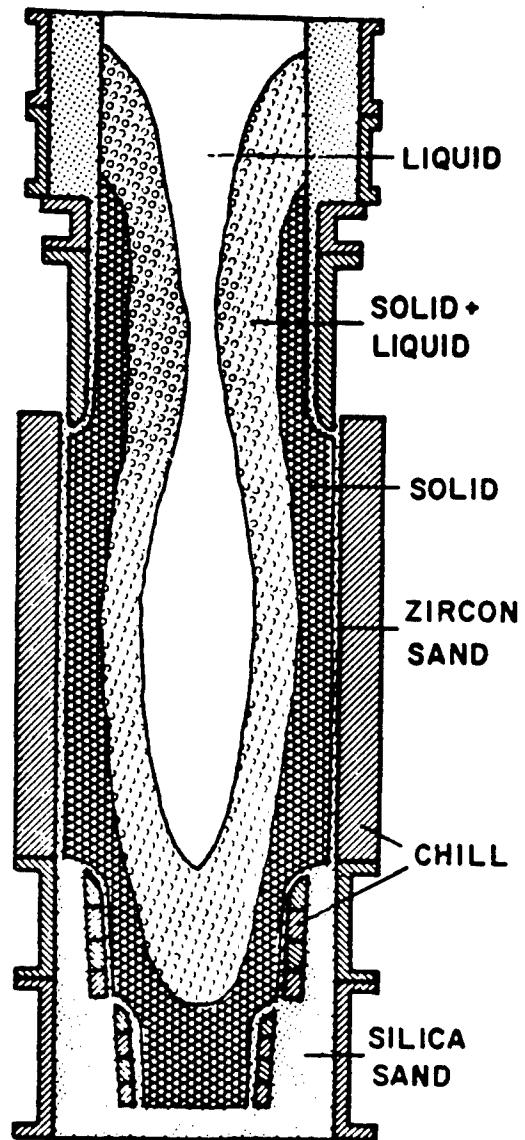


Figure IV-6: Large steel roll used for simulation. Time: 4 hours.

dimensional, axisymmetric implicit finite difference technique taking into account the temperature variation of the thermal properties of the alloy, using constant thermal properties for the facing sand, mold sand, and chills, taking into account thermal resistance at the interfaces and the introduction of an external heat source at the top of the cope neck. For the hypothetical mold design shown in Figures IV-6 and IV-7 (a design not used in practice), the computer simulation would predict gross shrinkage in the roll body below the cope neck.

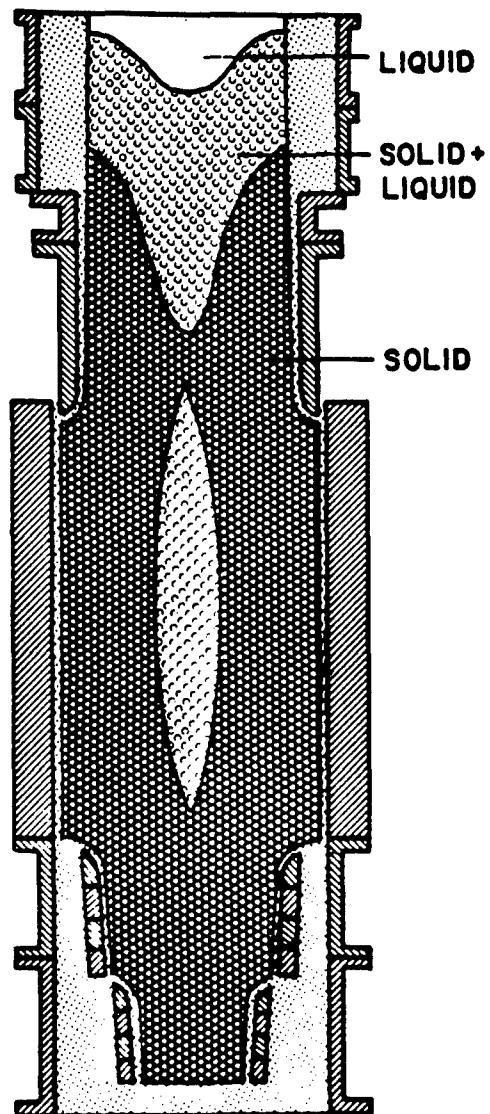


Figure IV-7: Large steel roll used for simulation. Time: 8.5 hours.

## IV.5 FEM FOR CONTINUOUS CASTING

The finite element method (FEM) is an alternative numerical technique for heat flow simulation, although it is receiving extensive application in the structural field for calculating loads, displacements, stresses, and strains. To illustrate its use in a heat flow application, consider DC continuous casting of round ingots. FEM was picked for this application so that it would be compatible with the thermal stress simulation done in the same program<sup>12,23,24</sup>.

FEM analyses have been used with a wider variety and complexity of elements than FDM analyses, although this use is not necessarily a limitation of the finite difference method.

In the finite element method, the mold and casting are first idealized and then divided into finite elements with nodes on the boundaries (primarily the corners) of the elements. The relations between position, thermal properties, and temperature within an individual element (element stiffness matrix) are determined. The relations for the individual elements are assembled into an overall relation (global stiffness matrix), keeping in mind each node should have a unique temperature (when calculated by the relations for any one of the elements sharing that node). The boundary conditions are applied to the external nodes (and the nodes on the centerline). The expressions relating temperature, the global stiffness matrix, and boundary conditions are solved (as a large group of simultaneous equations) to give the temperatures of each node.

The elements selected for the simulation of DC casting are shown in Figure IV-8. In cross section the elements are triangular; however, in relation to the cylindrical ingot the elements are rings. Two triangular elements make a quadrilateral elements. In the simplest, first-order finite element analysis, a node is located on each vertex of the triangle. Each node is shared by six triangular elements and four quadrilateral elements.

Within each element the temperature is considered to be a linear function of the temperatures at the nodes. One method to write the temperature within the element is to use interpolation functions

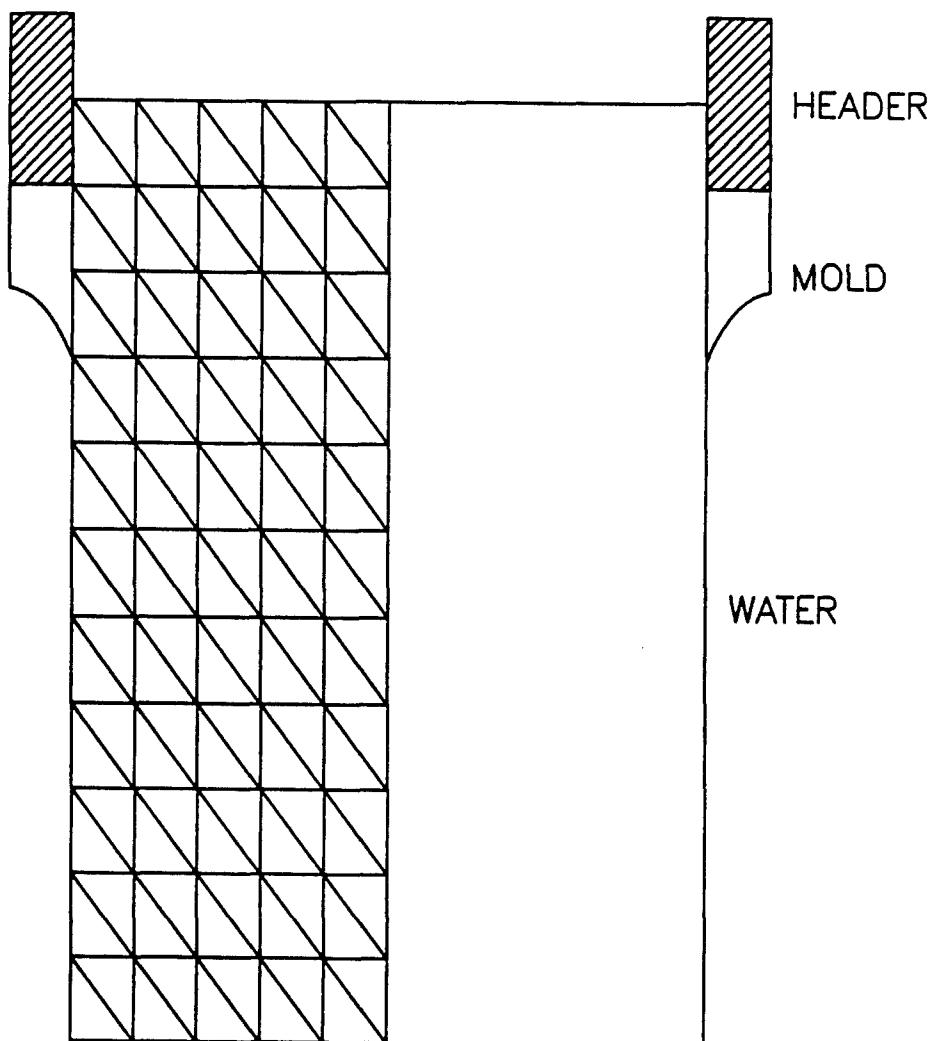
$$T = N_i T_i + N_j T_j + N_k T_k \quad (IV.14)$$

where the interpolation functions

$$N_i = 1 \text{ at node } i$$

$$N_i = 0 \text{ at nodes } j, k, \text{ etc.}$$

$$\sum N_i = 1 \text{ within the element}$$



R ←  
Z ↓

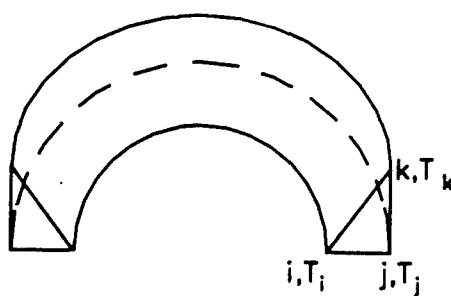


Figure IV-8: Triangular ring elements used in FEM for DC casting.

The basic equation for heat conduction given in Eqn. (IV.1) is modified for DC casting. A cylindrical coordinate system fixed with respect to the caster is selected ( $r = 0$  at the centerline,  $z = 0$  at the meniscus). To account for motion of the ingot a term such as the second term on the right of Eqn. (IV.1) is required. Considering the caster to have reached steady operation, the first term on the left of Eqn. (IV.1) becomes zero: thus,

$$0 = \frac{1}{r} \left\{ \frac{\partial}{\partial r} (kr \frac{\partial T}{\partial r}) + \frac{\partial}{\partial z} (kr \frac{\partial T}{\partial z}) \right\} - \rho C_p v \frac{\partial T}{\partial z} \quad (\text{IV.15})$$

The Galerkin weighted residual method<sup>16</sup> is applied to Eqn. (IV.14) and (IV.15).

$$2 \pi r \int_V \left( \kappa \frac{\partial T}{\partial r} \frac{\partial N_i}{\partial r} + \kappa \frac{\partial T}{\partial z} \frac{\partial N_i}{\partial z} + \rho C_p v N_i \frac{\partial T}{\partial z} \right) dr dz - \int_S \left\{ \kappa \frac{\partial T}{\partial r} I_r N_i + \kappa \frac{\partial T}{\partial z} I_z N_i \right\} dS = 0 \quad (\text{IV.16})$$

In assembling these for the entire casting, the surface integrals will cancel on neighboring elements and will give a contribution only for the boundary elements. The individual relations are assembled into a global set of equations:

$$[H] \{T\} = \{F\} \quad (\text{IV.17})$$

where

$[H]$  is the global stiffness matrix.

$\{T\}$  is the vector (column matrix) of all nodal temperatures in the body.

$\{F\}$  is the thermal load vector (contributions from all the boundaries).

The equations are solved using an iteration technique for  $\{T\}$ .

A wide variety of boundary conditions can be used in continuous casting simulation. For example, the centerline is considered adiabatic; the alloy enters at a constant superheat; the heat transfer between alloy and mold is initially good and then becomes poor as the solid skin shrinks away from the mold, leaving the mold the ingot experiences a high heat transfer coefficient in the water blanket. The position of air gap formation may be calculated by the program.

Examples of the results of the thermal analysis are shown in Figure IV-9 and Table IV-1. Descriptions of the validation procedures and further applications of

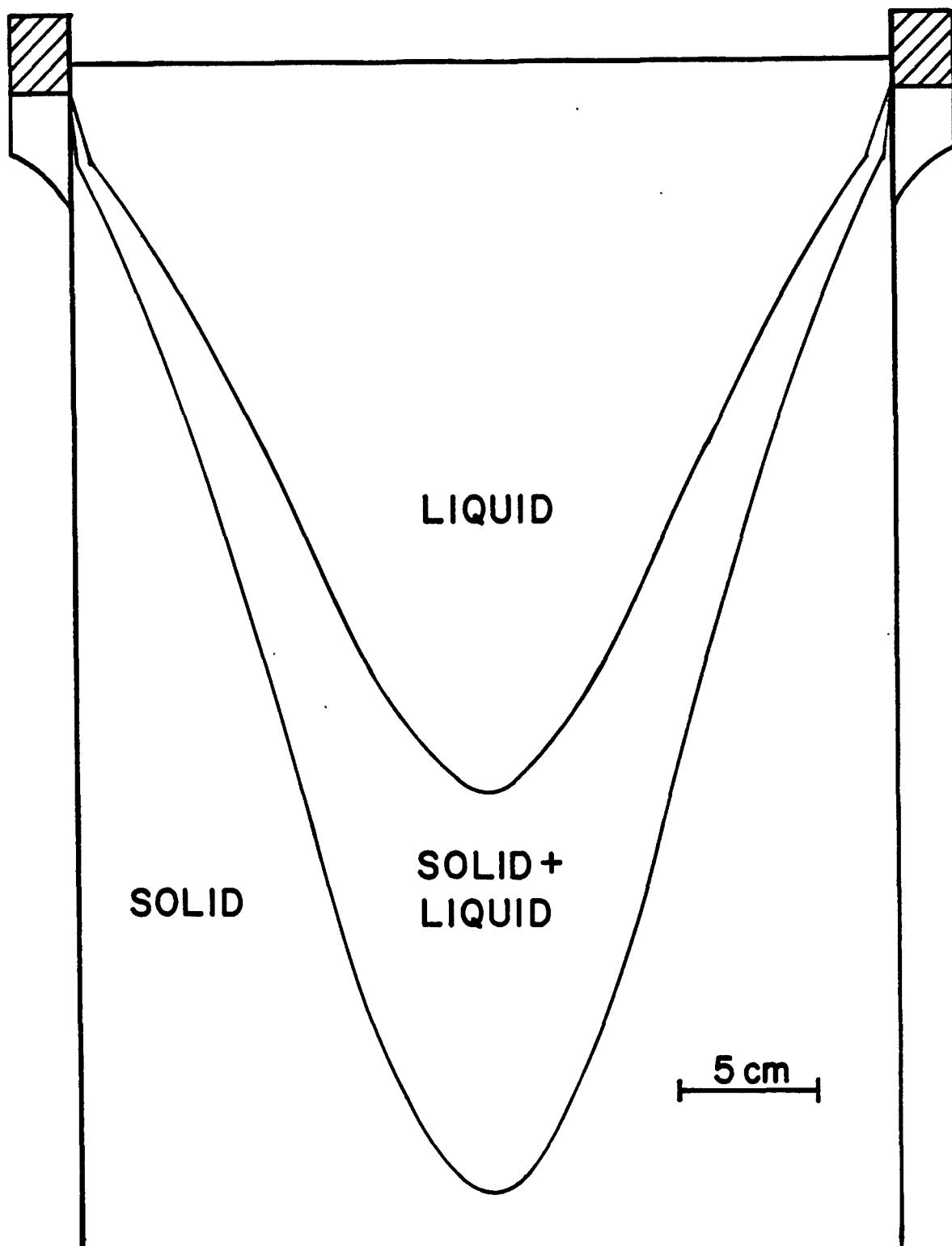


Figure IV-9: Liquidus and solidus isotherms, 38 cm  $\phi$ , 10 cm/min.

this and other models are given in the references<sup>13,25-27</sup>. Further, the results of the thermal analyses have been used as the bases of finite element analyses of the development of thermal stresses in nonferrous DC casting<sup>13</sup> and continuous casting of steel<sup>27</sup>.

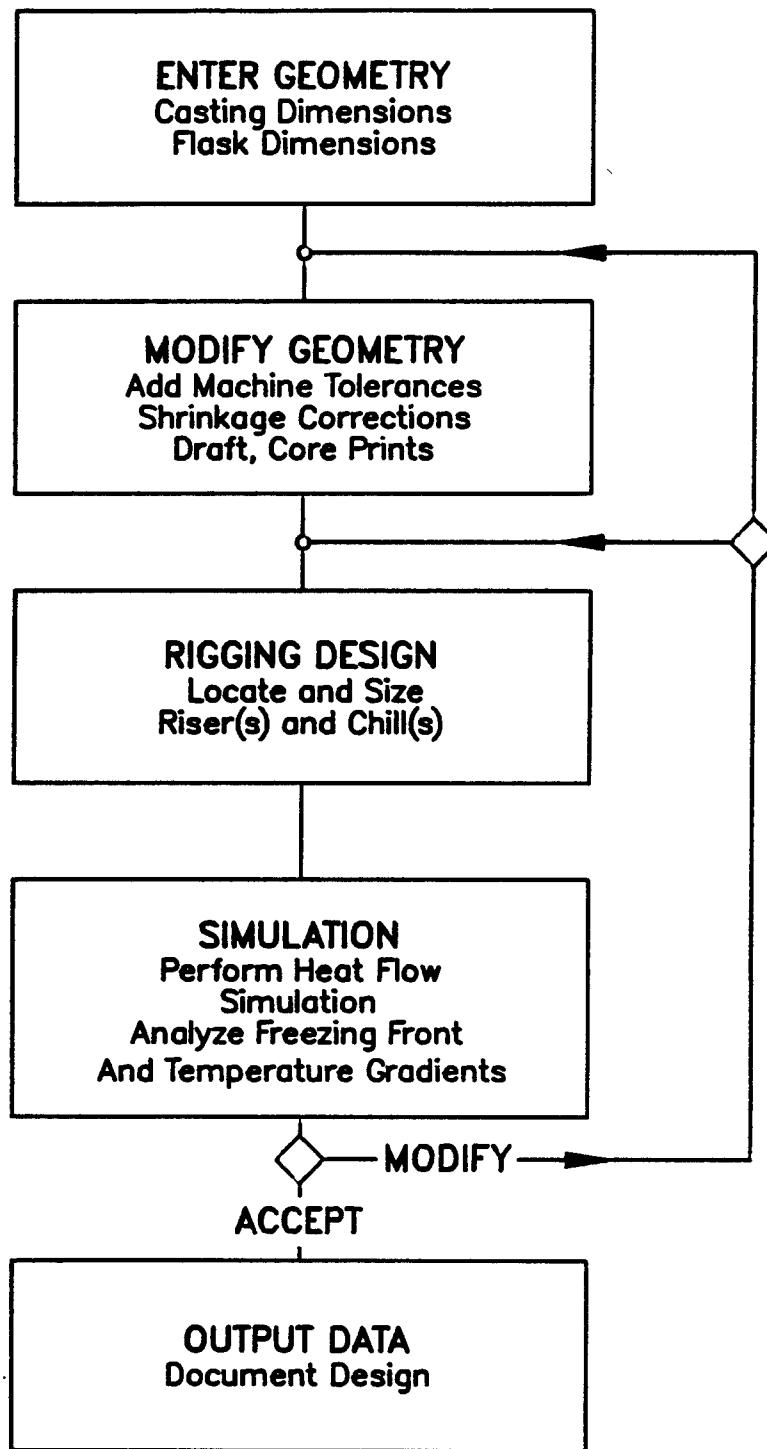
**Table IV-1: Effect of Secondary Cooling on Heat Flux in D C Casting.**

Distance from Meniscus, cm	Heat Transfer Coefficient in the Water Blanket, cgs		
	0.05	0.60	0.80
Heat Flux (cgs)			
1.25	32	33	34
3.75	9	1.7	1.6
6.25	23	93	102
8.75	21	36	34
11.25	19	30	30
13.75	18	24	24
16.25	17	21	

\* 38 cm Aluminum alloy ingot cast in a 5 cm mold at 10 cm/min.

## IV.6 FEM ANALYSIS OF SAND CASTINGS

A similar FEM analysis may be made of sand castings as part of an overall mold design process. First, the geometric description of the part to be cast would be entered into the computer; then normal casting design methods<sup>28,29</sup> would be used to place and size risers. At this point, when a pattern and other tooling would normally be made, the computer program would allow the thermal simulation of the freezing of the casting. Here it would be desirable to seek interpretation of the result further than that described above with respect to large roll castings. Even though the freezing pattern in a casting would be progressive to the risers, the casting might not be sound.<sup>29</sup> Thus, criteria such as thermal gradients in critical regions of the castings would have to be evaluated in order to predict soundness. A simplified flow diagram is shown in Figure IV-10.



**Figure IV-10:** Procedure for computer aided design & simulation of castings.

## IV.7 CONCLUSION

The techniques illustrated above and others are available for simulating casting processes. While heat flow analysis was used as an example, other macroscopic analyses may be carried out readily. The numerical techniques are being made suitable for casting process simulation as a result of the current expansion of effort. General development in computer graphics and geometric modeling will ease the task of entering data, especially geometric data, generation of element meshes, and analysis of results. The expected increase in speed and efficiency of machine computation and memory capacity will make more complex simulations practical and will be definite advantages in adapting simulations to real time process control and process design (CAD/CAM). To take advantage of the first developing capability in simulation, more effort should be directed to applying simulations to the casting process for both design and control. This will require learning more about the processes, especially on a quantitative basis, and measuring alloy mechanical, transport, and thermal properties in the temperature range of interest for casting processes. Success in applying the models in practice will be the impetus for making these difficult measurements.

## ACKNOWLEDGEMENTS

Illustrative examples in this paper were taken from work sponsored by the Office of Naval Research Metallurgy Program, Contracts N00014-75-C-0800 and N00014-77-C-0596, Dr. Bruce MacDonald, Scientific Officer; by the Roll Manufacturers Institute, Project No. 41, Dr. Robert Corbett, Technical Director; and by the U.S. Army Tank and Automotive Research and Development Command, DAAK30-78-C-107, Mr. Thomas Wassel, Technical Officer.

## REFERENCES

1. Modeling of Casting and Welding Processes, eds., H. D. Brody and D. Apelian, T.M.S.-A.I.M.E., Warrendale, PA., 1980.
2. G. H. Gulliver, "The Quantitative Effect of Rapid Cooling Upon the Constitution of Binary Alloys," J.I.M., 9 (1913) p. 120-157.
3. E. Schiel, "Bemerkungen zur Schichtkristallbildung," Z. Metallk., 34 (1942) p. 70-72.
4. H. D. Brody and M. C. Flemings, "Solute Redistribution in Dendritic Solidification," Trans. AIME, 236 (1966) p. 615-614.
5. T. F. Bower, H. D. Brody, and M. C. Flemings, "Measurement of Solute Redistribution in Dendritic Solidification," Trans. AIME, 236 (1966) p. 625-634.
6. M. C. Flemings, R. V. Barone, D. R. Poirier, and H. D. Brody, "Microsegregation in Iron Base Alloys," J.I.S.I., 208 (1970) p. 371-381.
7. J. I. Nurminen and H. D. Brody, "Dendrite Morphology and Microsegregation in Cast Titanium Alloys," Titanium Science and Technology, Plenum (New York, 1973) p. 1893-1914.
8. L. A. Lalli, "The Effect of Dendrite Migration on Solute Redistribution," in reference 1.
9. G. E. Nero and M. C. Flemings, "Macrosegregation, Part I," Trans. AIME, 239 (1967) p. 1449-1459.
10. E. T. Turkdogan, "Deoxidation of Steel - What Happens from Tap to Solidification," J. of Metals 19 (1962) p. 38-44.
11. R. H. Tien and G. E. Geiger, "A Heat Transfer Analysis of the Solidification of a Binary Eutectic System," Trans. - A.S.M.E. - J. Ht. Trans., 89 (1967) p. 230.
12. J. Mathew and H. D. Brody, "Analysis of Heat Transfer in Continuous Casting Using Finite Element Method," Computer Simulation for Materials Applications, Nuclear Metallurgy, ed. by R. J. Arsenault et al. vol. 20. Part 2, (1976) p. 1138-1150.

13. J. Mathew and H. D. Brody, "Simulation of Thermal Stresses in Continuous Casting Using a Finite Element Method," Computer Simulation for Materials Application, Nuclear Metallurgy, ed. by R. J. Arsenault et al. vol. 20, part 2, (1976) p. 978-990.
14. H. D. Brody, "Simulation of Solidification Processes," Process Modeling Symposium, A.S.M. Philadelphia, (1978) in press.
15. G. F. Forsythe and W. R. Wasow, Finite-Difference Methods for Partial Differential Equations, John Wiley and Sons (New York, 196).
16. C. S. Desai and J. F. Abel, Introduction to the Finite Element Method Van Nostrand Reinhold Co. (New York, 1972).
17. H. R. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford Press (London, 1959).
18. M. C. Adams, Jr., "Thermal Considerations in Freezing," Adams, Jr., "Thermal Considerations in Freezing," p. 187-217 in Liquid Metals and Solidification, A.S.M. (Cleveland, 1958) p. 187-217.
19. A. Tzavaras, M. Vasiliaros, H. D. Brody, and G. M. Ecer, "Effects of Welding Parameters on Solidification Structure in Pulsed G.T.A. Welded Fe-26Ni," Physical Metallurgy of Welding ed. by R. Kossowsky and M. Gilicksman; TMS-AIME, (Warrendale, Pennsylvania, 1980).
20. G. M. Ecer, G. G. Lessman, and H. D. Brody, "Investigation of Weld Pool Structure and Property Control in Pulsed Arc Welding," Westinghouse, Annual Report to Office of Naval Research, Contract N00014-77-C-0596, (October 1978).
21. G. M. Ecer, G. G. Lessman, and H. D. Brody, "Investigation of Weld Pool Structure and Property Control in Pulsed Arc Welding," Westinghouse, Annual Report to Office of Naval Research, Contract N00014-77-C-0596, (November 1979).
22. R. A. Stoehr and H. D. Brody, "Solidification Contours of Selected Rolls," Roll Manufacturers Institute, Project No. 41A, (March 1976).
23. R. W. Southworth and S. L. Deleeuw, Digital Computation and Numerical Methods, McGraw-Hill (New York, 1965).

24. J. Mathew and H. D. Brody, "Simulation of Heat Flow and Thermal Stress in Axisymmetric Continuous Casting," Solidification and Casting of Metals, Metals Society (London, 1979) p. 244-249.

25. E. A. Mizikar, "Mathematical Heat Transfer Model for Solidification in Continuously Cast Steel Slabs," Trans. AIME, 239 (1967) p. 1747-1753.

26. A. Grill, J. K. Brimacombe, and F. Weinberg, "Mathematical Analysis of Stresses in Continuous Casting of Steel," Iron & Steelmaking, 76 (1976) p. 38-47.

27. J. K. Brimacombe and F. Weinberg, "Continuous Casting of Steel -- Part 2: Theoretical and Measured Liquid Profiles in the Mold Region During the Continuous Casting of Steel," J.I.S.I., 211 (1973) p. 24-33.

28. R. Wlodawer, Directional Solidification of Steel Castings, Pergamon Press (Oxford, 1966).

29. H. F. Bishop, E. T. Mykowski, and W. S. Pellini, "A Simplified Method for Determining Riser Dimensions," A.F.S. Trans., 63 (1955) p. 271-281.

## LIST OF SYMBOLS

$C_p$	=	heat capacity, cal/g °C.
$e_b, e_p$	=	arc potential for background and peak pulses, volts.
{F}	=	vector of boundary conditions for FEM thermal analysis.
[H]	=	global thermal stiffness matrix for FEM.
$i_b, i_p$	=	arc current for background and peak pulses, amperes.
I,J,K,L	=	subscripts to index node and element positions.
$l_x, l_z$	=	direction cosines.
M,M'	=	moduli, Eqn. (IV.6), for explicit and implicit FDM, respectively.
$N_i$	=	interpolation functions for FEM.
Q	=	heat input from external or internal heat source, cal/ s•cm <sup>3</sup> .
$q_b, q_p$	=	heat input from arc during background and peak pulses, cal/s.
R	=	linear velocity of bulk alloy flow, cm/s.
r	=	radial distance in cylindrical coordinates, cm.
T,T'	=	instantaneous temperature, and temperature before and after time step, °C.
t,t <sub>b</sub> ,t <sub>p</sub>	=	time variable and duration of background and peak pulses, s. Also, t is used for sheet thickness of weldment, cm.
$\Delta t$	=	time step, s.
v	=	arc travel speed and casting rate, cm/s.
X	=	distance in heat flow direction, cm.
$\Delta X$	=	general finite element node spacing, cm.
$\Delta X_L, \Delta X_S$	=	node spacing in large and small grids of welding FDM, cm.
z	=	axial distance in cylindrical coordinates, cm.
$\alpha$	=	thermal diffusivity Eqn. (IV.2), cm <sup>2</sup> /s.
$\kappa$	=	thermal conductivity, cal/°C•cm <sup>2</sup> •s.
$\rho$	=	density, g/cm <sup>3</sup> .

## V. ANALYSES OF SAND AND INVESTMENT CASTING

Harold D. Brody

Materials Science and Engineering  
University of Pittsburgh  
Pittsburgh, Pennsylvania 15261

### ABSTRACT

Heat transfer models based on finite element and finite difference techniques have been applied to the freezing of simple and complex shaped castings. Evaluation of techniques has been in terms of suitability for use by foundrymen and accuracy of representation. Thermal measurements made on low alloy steel castings poured in sand molds have been used to calibrate the numerical models.

Fluid flow in mold cavities and gating systems has been analyzed by a variety of techniques. Simple analyses have been implemented to predict filling rate and heat losses in gating systems. Further analyses have been aimed at predicting distribution of metal in branched gating systems. A finite difference technique, the Marker and Cell technique, has been used to predict the "open channel" flow pattern of alloys in simple and complex shaped mold cavities. A planned extension would be to combine the heat flow and fluid flow analyses.

Measuring the thermal and flow properties needed as input in sand and investment casting presents a challenge that must be met if the models are to be successfully implemented.

\* Originally presented and published: Harold D. Brody, "Numerical Analyses of Sand and Investment Casting," Symposium on Solidification Science, sponsored by the National Science Foundation at the Massachusetts Institute of Technology, July 1982.

## V.1 INTRODUCTION

Recently, major efforts have been applied to making rigorous analyses of casting processes and to making these analyses available to foundrymen for use in design of the casting process for complex shapes. Numerical techniques when implemented on an electronic computer can be used to simulate the behavior of large and complex castings within practical times and costs and with reasonable accuracy. Certainly, the analyses can be used to show trends and the effects of changes in process variables. The techniques for analysis of heat flow are well developed. Fluid flow and stress analyses are not as well developed for simulation of castings, although significant work is underway. The difficulty in bringing these analyses into use by foundrymen in designing the rigging of a casting (determining size and placement of risers, sizes and numbers of runners, sprues, and ingates, need and placement of chill and insulating materials and padding. Flask size and orientation of the pattern in the flask) includes the following:

1. Making the analyses "user friendly" to be used by foundrymen without either computer programming experience or detailed knowledge of the numerical analysis techniques.
2. Optimizing the numerical analysis technique and the computer program to make efficient use of computer memory and running time.
3. Determining the appropriate material properties (usually at high temperature) and casting parameters to the degree of accuracy desired.
4. Developing criteria that relate casting soundness to the basic process parameters amenable to computer simulation in a form suitable for use by foundrymen.

Some of the progress in simulation of casting and the potential for their use in design can be demonstrated by considering the analysis of a stepped plate casting. This is a rather simple shape; however, it has been used extensively as a test casting<sup>1,2</sup> and it illustrates clearly many of the complexities that must be included. Patterns for two stepped plate castings in sand molds are shown in Figure V-1. Plate A has four steps and should be a sound casting with a single end riser. Plate D has two steps and the 20" X 7" X 2" step should be unsound with the riser configuration shown.

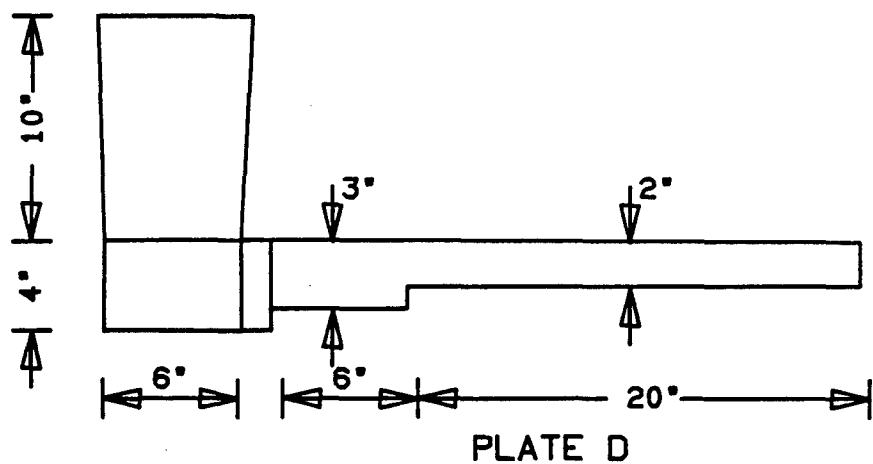
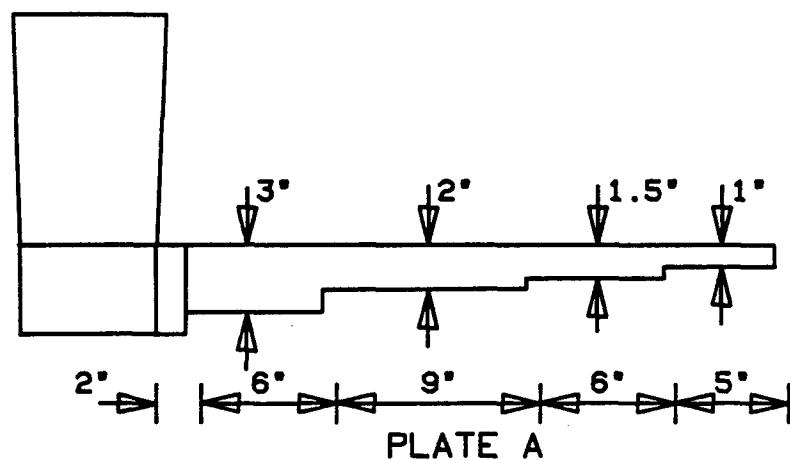
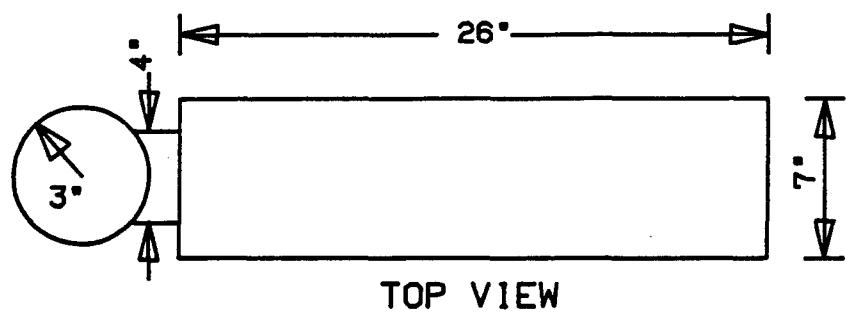


Figure V-1: Configurations of stepped plate test castings.

## V.2 HEAT FLOW

Finite difference and finite element methods have been applied to the analysis of heat flow in castings<sup>2-6</sup>. The analyses have been applied with good results to castings of substantial complexity. However, these simulations have been implemented by experts in heat flow analysis and usually on main frame computers.

### V.2.1 Numerical Analysis Methods

The general approach is to evaluate the basic equation for heat flow by conduction:

$$\rho C_p (\partial T / \partial t) = \nabla \cdot (k \nabla T) \quad (V.1)$$

where

$\rho$  = density

$C_p$  = heat capacity which may be modified to include the heat of fusion

$k$  = thermal conductivity which may be modified for the liquid to account for convection

$T$  = temperature at some time and position within the alloy or mold

$t$  = time

$\nabla T = \partial T / \partial X + \partial T / \partial Y + \partial T / \partial Z$

Note that a convective term has not been included. By making justifiable assumptions, appropriate initial and boundary conditions can be specified.

The common initial condition is based on the assumption that the alloy has entered the mold at temperature  $T_{pour}$  and is stagnant before beginning to cool. Similarly, the mold is assumed to remain at temperature  $T_{mold}$  until the time the mold is filled. This assumption appears to be adequate for castings of medium to large cross section. It would be unreasonable in the case of thin castings. There are schemes for approximating the effect of mold filling on heat transfer that do not increase greatly, the complexity of the analyses. True coupling of the heat flow and fluid flow analyses has not been accomplished.

The external boundary conditions for sand molds has simply been convective heat transfer

$$h (T_b - T_o) + k (\partial T / \partial n) = 0 \quad (V.2)$$

where

$h$  = heat transfer coefficient

$T_b$  = mold temperature at the boundary at the time of evaluation

$T_o$  = ambient temperature - assumed constant

$n$  = distance vector normal to boundary.

In the case of sand molds, the mold walls are sufficiently thick and the sand sufficiently insulating that the analysis is not sensitive to the assumptions made at the external boundaries. For thin walled molds, e.g. shell molds, more care is required and heat transfer by thermal radiation may have to be included. Internal boundaries between alloy and mold materials, may be considered as completely conducting for normal insulating mold materials, e.g. silica sand, plaster, and investment coats. When the alloy is in contact with chill material, consideration of an interface heat transfer coefficient is required. Measurement of the interface heat transfer coefficient presents a problem including the need to properly take into consideration the gap formed when the casting shrinks away from the mold wall. Boundary conditions such as equation (V.2) would be applied in numerical analyses where where  $h$ ,  $T_0$ , and  $T_b$  could vary with time.

Discussion of the finite difference (FDM) and finite element (FEM) methods of evaluating these equations<sup>3</sup> is beyond the intention of this paper. In both methods space and time are discretized. Temperature varies continuously with time and distance (except at some boundaries) in a real casting. In both F.D.M. and F.E.M. space is divided into contiguous regions called elements and representative nodes are located within or on the edges of the elements. In F.D.M. the temperature is assumed to vary in finite steps from element to element and to vary in finite steps with time. In F.E.M. the temperature is assumed to vary by a simple function (e.g. linearly) within an element and temperatures are matched at nodes common to two or more elements. Temperature varies in finite steps with time.

As example, Equation (V.1) in a finite difference technique could be rewritten for one dimension

$$\rho C_p \cdot \frac{T(X,t+\Delta t) - T(X,t)}{\Delta t} = \frac{T(X+\Delta X,t) - T(X,t)}{\Delta X} - \frac{T(X,t) - T(X-\Delta X,t)}{\Delta X} \quad (V.3)$$

where

$\Delta t$  = size of the time increment

$\Delta X$  = distance between element centers.

In algorithm (V.3)  $T(X,t+\Delta t)$  is the unknown, everything else is known. The values of the material properties, if known as a function of temperature, can be inserted with the values appropriate to  $T(X,t)$ . Thus, it is an easy matter to take into account the heat evolution during freezing by modifying the heat capacity to include the heat of fusion distributed judiciously over the freezing range. In most cases measured in our laboratory, the evolution of latent heat is concentrated in a small temperature range close to the liquidus temperature.

The observed variability in molding sand properties from mold to mold and day to day in commercial steel foundry operation led us to not use temperature dependent thermal properties for molding sand.

The results of a 2-D analysis of the heat flow in test stepped plate casting A and the results of experimental measurements on armor steel cast into a no-bake sand mold for plate A are compared in Figure V-2. The 2-D analysis is made for the central (symmetry) plane for the stepped plate casting. The simulated cooling curves and the measured cooling curves agree superficially. The computed results could be used to predict whether the casting would freeze progressively from thin end to the riser. However, closer inspection shows severe discrepancies between predicted and measured cooling rates. Good agreement is not possible with a 2-D analysis.

The results of a 3-D F.E.M. heat flow analysis are compared to the experimental data in Figure V-3. In this analysis half of the casting and mold were analyzed taking advantage of the symmetry about the central plane. The agreement of the experimental and computed cooling curves in both thick and thin sections is good. Thus, if an F.E.M. analysis is to be applied to even this simple casting, a 3-D analysis is required to predict cooling curves.

### V.2.2 Application to Casting Process Design

It should be pointed out that the 2-D analysis of plate A was based on a grid of 720 nodes and the 3-D analysis was based on analysis of 4400 nodes. This points up the need for automatic means of automatically generating the nodes and elements required for numerical analysis of a complex shape.

The use of the heat transfer analysis in design may not require as much accuracy as is required to get good prediction with cooling curves. However, the ability to use the analysis to predict the detailed behavior as measured in simple cases gives confidence that the analysis may be extrapolated to more complex cases.

One obvious criterion for use with heat flow analyses in design is that the freezing front should move progressively toward the riser. After a heat flow analysis has been done and mounds of predicted results accumulated, the position of selected isotherms superimposed on an image of the casting's cross section can be displayed on a terminal screen. This allows ready visual inspection of the results of the simulation and their use in design, especially if successive positions of the isotherms are shown on the screen and hardware is available for color representation.

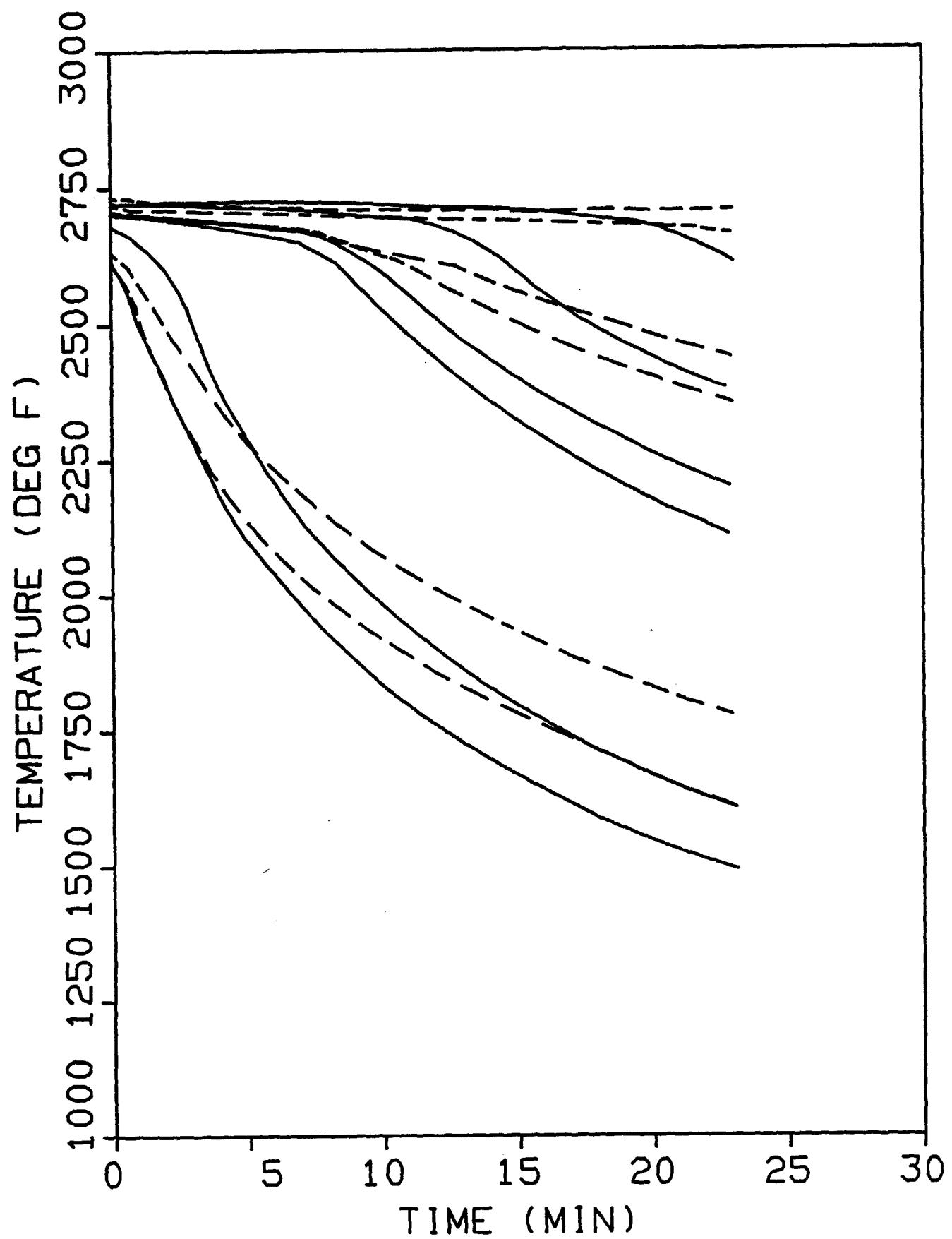


Figure V-2: Cooling curves, Plate A. (a) 2-D simulation (dashed).  
(b) Measured (solid).

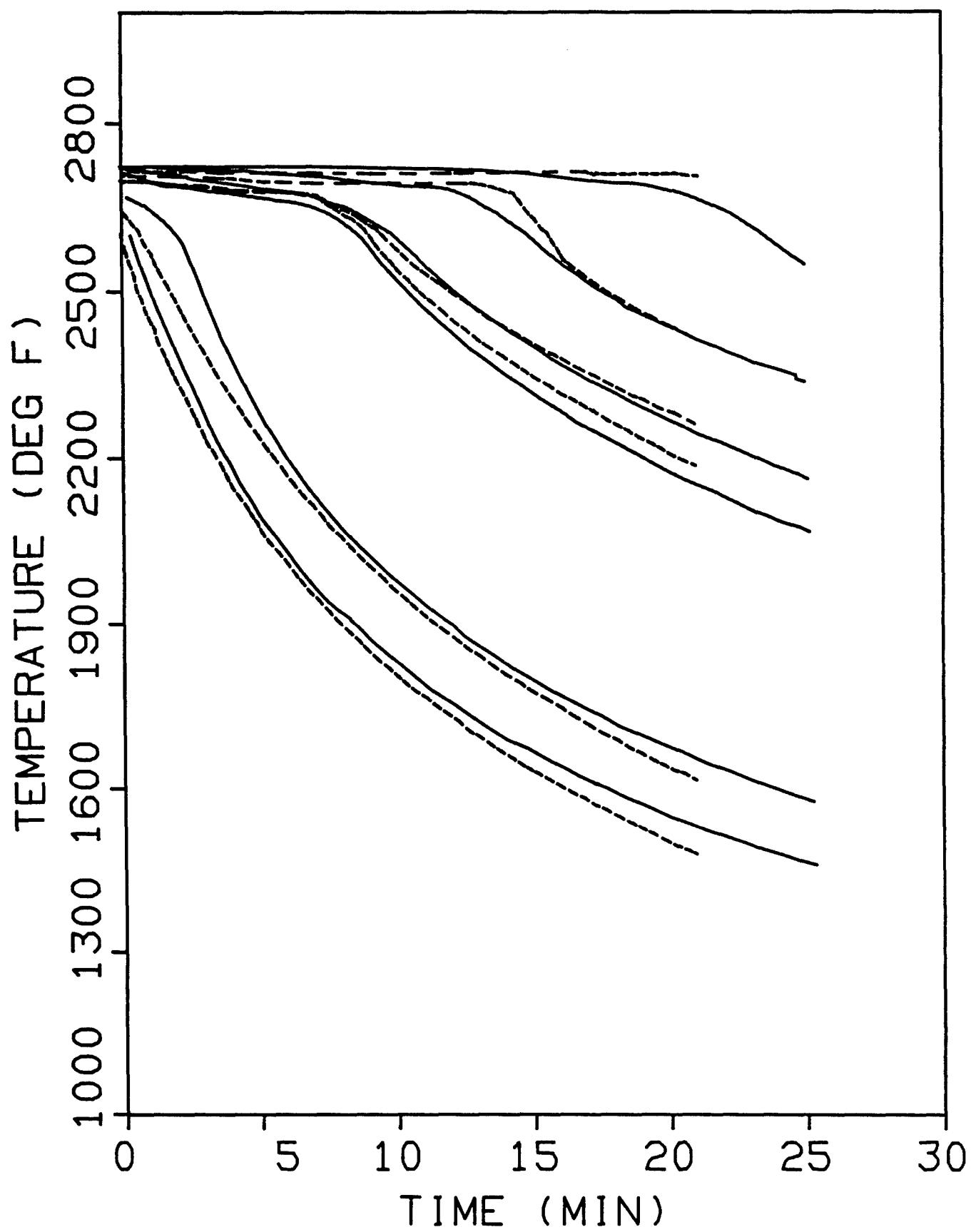


Figure V-3: Cooling curves, Plate A. (a) 3-D simulation (dashed).  
(b) Measured (solid).

Progressiveness of freezing is not a sufficient criterion to insure the soundness of a casting. The temperature gradients during freezing must be sufficiently steep to allow flow of liquid through the mushy zone to feed shrinkage. The concept is not new<sup>1,7,8</sup>. Because it has not been practical to predict thermal gradients in critical sections of castings, there has been little incentive to measure the critical thermal gradients for achieving a desired level of soundness. The potential application of computer simulation to design makes use of this criterion practical and its determination worthwhile.

Considering freezing of plate A and plate D, experimental and computer temperature data indicate both freeze progressively. Nonetheless plate A will be radiographically sound and plate D will exhibit sufficient centerline shrink for it to be apparent in radiographs. Comparison of the measured temperature gradient during freezing of plates A and D, as in Figure V-4, demonstrates the actual difference in minimum temperature gradient during freezing. That radiography of plate D indicated porosity at the position of low temperature gradient illustrates the utility of this criterion.

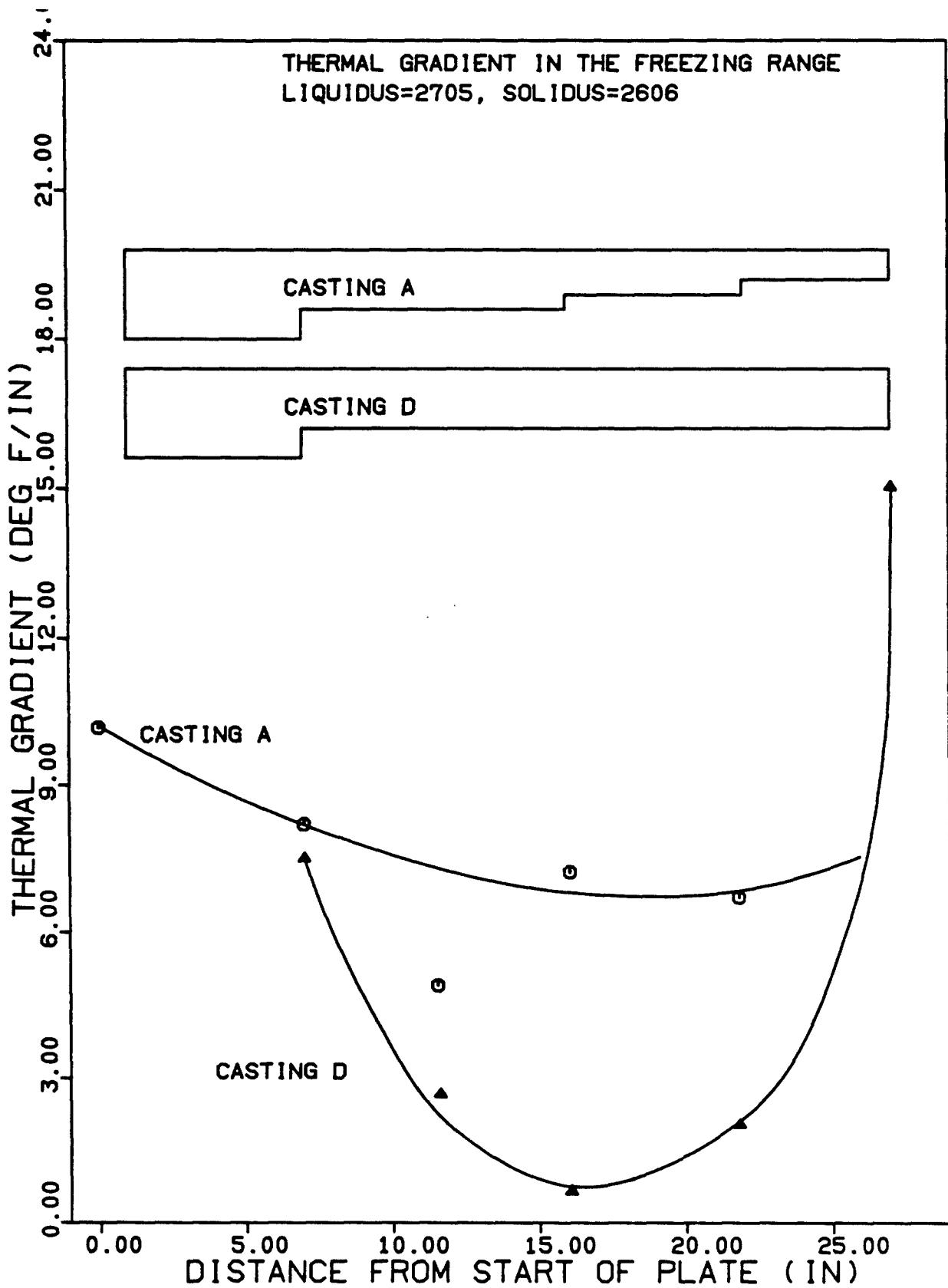


Figure V-4: Thermal gradient during freezing, centerline, Plates A & D.

## V.3 FLUID FLOW

Flow of liquid metal in the gating system and mold cavity associated with the filling of the mold may be as important to the soundness of a casting as riser placement. The influence of gating on initial heat distribution in the mold and alloy was mentioned earlier. The pattern of flow influences the ability to fill thin sections, the formation and trapping of inclusions and gas, the erosion of mold materials, uniform delivery of metal through ingates, and the location of cold shuts.

Once channels are full Bernoulli's equation may be used to describe the flow<sup>8</sup>. This may be used to establish filling time, uniformity of filling in a multigate system, and heat loss in the gating system<sup>9,10</sup>. Simulation of the pattern of filling of mold cavities requires analysis of open channel flow. This will allow prediction of order of mold filling, formation of vortices, local high pressure regions, and local metal velocity.

### V.3.1 Marker and Cell Technique

One method of analysis of open channel flow is the marker and cell (MAC) technique (and the simplified marker and cell technique - SMAC)<sup>11,12</sup>. For a 2-D analysis the basic equations are the continuity equation

$$\frac{\partial u}{\partial X} + \frac{\partial v}{\partial Y} = 0 \quad (V.4)$$

and the momentum equations in X and Y directions

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial X} + \frac{\partial uv}{\partial Y} = -\frac{\partial \Phi}{\partial X} + v \left( \frac{\partial^2 u}{\partial X^2} + \frac{\partial^2 u}{\partial Y^2} \right) + g_x \quad (V.5)$$

$$\frac{\partial v}{\partial t} + \frac{\partial v^2}{\partial Y} + \frac{\partial uv}{\partial X} = -\frac{\partial \Phi}{\partial Y} + v \left( \frac{\partial^2 v}{\partial X^2} + \frac{\partial^2 v}{\partial Y^2} \right) + g_y \quad (V.6)$$

where

$u$  and  $v$  = the local linear velocities in the X and Y directions

$\Phi$  = reduced pressure (divided by density)

$v$  = kinematic viscosity

The channel is divided into a grid of elements (called cells) and nodes and equations (V.4) - (V.6) are written in finite difference form. Markers are considered to enter the channel along with the fluid and accounting is kept of the position of each of the markers after each time step. Local pressures and velocities are also recorded. Again graphic representations are a convenient way of reviewing results of the simulation.

### **V.3.2 Filling of a Vertical Plate**

Consider the filling of a vertical plate shaped cavity from a gate on the side at the bottom. The velocity vectors are depicted for six different times during filling on Figure V-5. The direction and magnitude of the vectors represent the local vector summation of the computed velocities in the X and Y directions (i.e.  $u + v$ ). The distribution of arrows indicates the pattern of filling. The alloy flows across the bottom and then builds up along the far wall before reflecting back toward the entrance, forming a wave that builds up and reflects from the near wall. Note the formation of a vortex in the region above the gate and the formation of a dead space in the bottom corner away from the gate. All of these observations agree with water model studies.

### **V.3.3 Effect of Filling Rate**

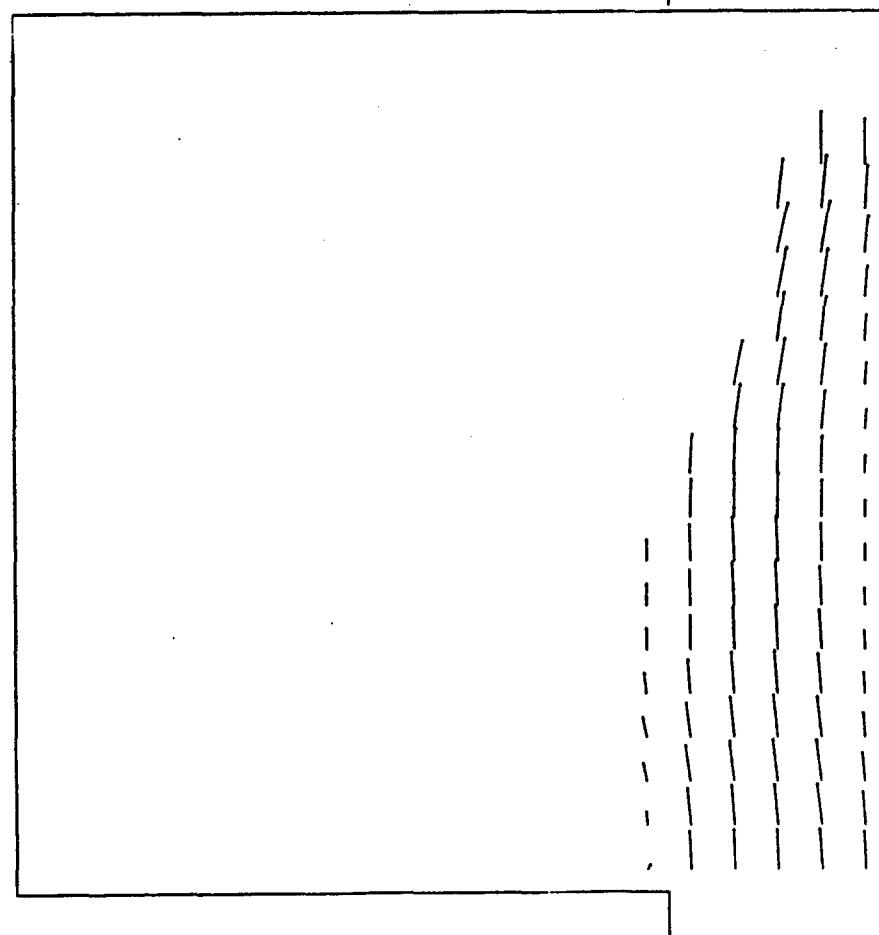
The filling of a horizontal plate from a single gate at the center of one side has been simulated with filling rate being one variable. The results of the simulations near the end of filling are shown for two filling rates in Figure V-6. Again the velocity vectors are shown. The top diagram simulated a filling rate comparable to sand or investment castings. In frames not shown the metal stream flowed across the center of the plate, expanding only slightly. When it reached the far wall it split into two streams building up along the far mold cavity wall and then reflected back toward the entrance. The returning stream and the entering stream generally merged. In Figure V-6 at the top it is shown that the last place to fill is adjacent to the ingate.

The lower condition in Figure V-6 simulates a much higher filling velocity comparable to die casting. Here the entering stream spouts across the center of the plate with very little expansion, and hugs the far wall, then the top and bottom (with reference to the orientation of the figure on the page) walls, and then the near wall before filling the center of the casting cavity. The last regions to fill and the flow pattern near the end of filling are shown in Figure V-6 bottom. Note the potential for gas entrapment in die casting.

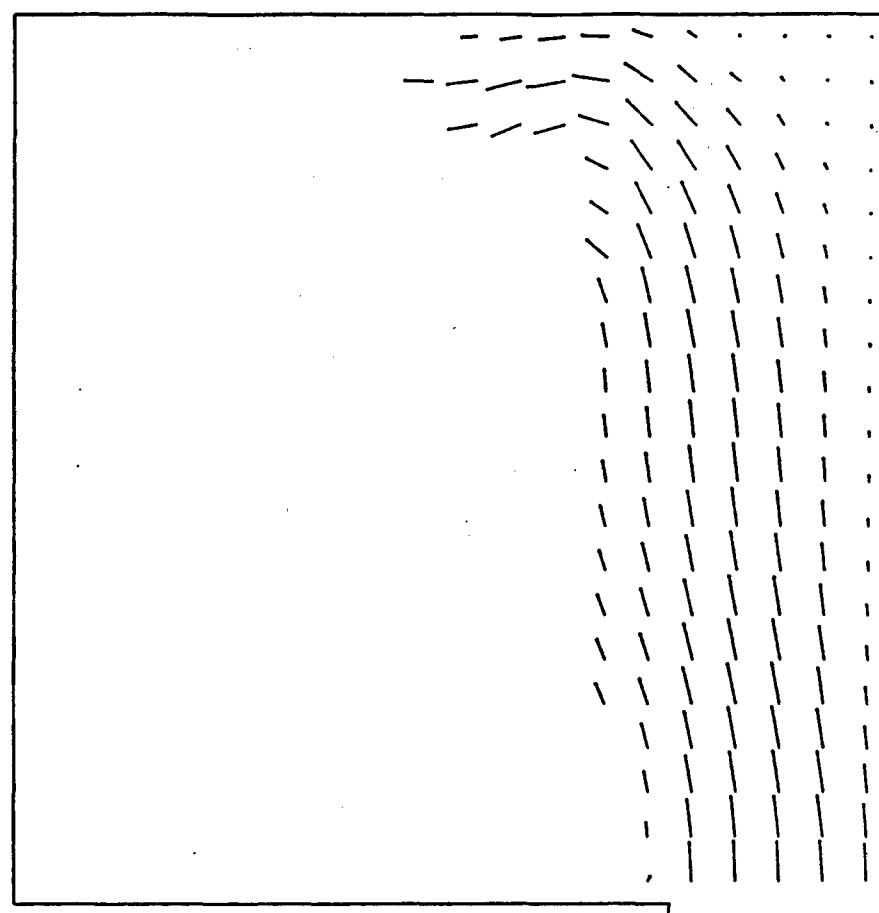
### **V.3.4 Filling of Plate A**

The marker and cell technique has been applied to the filling of a stepped plate casting, as shown in Figure V-7. Note the surges when the metal enters each step, that the riser substantially fills before the plate is filled, and that local vortices form in the riser pad and riser.

TIME: 0.11

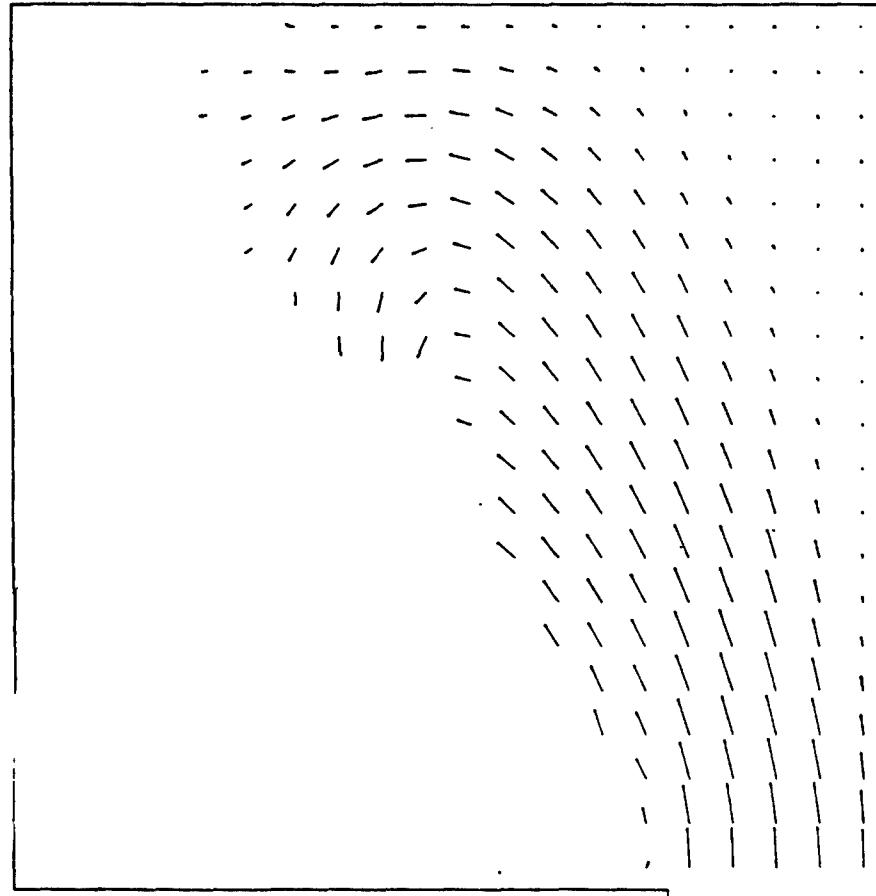


TIME: 0.22



**Figure V-5:** Velocity vectors computed for filling of a vertical plate.

TIME: 0.33



TIME: 0.44

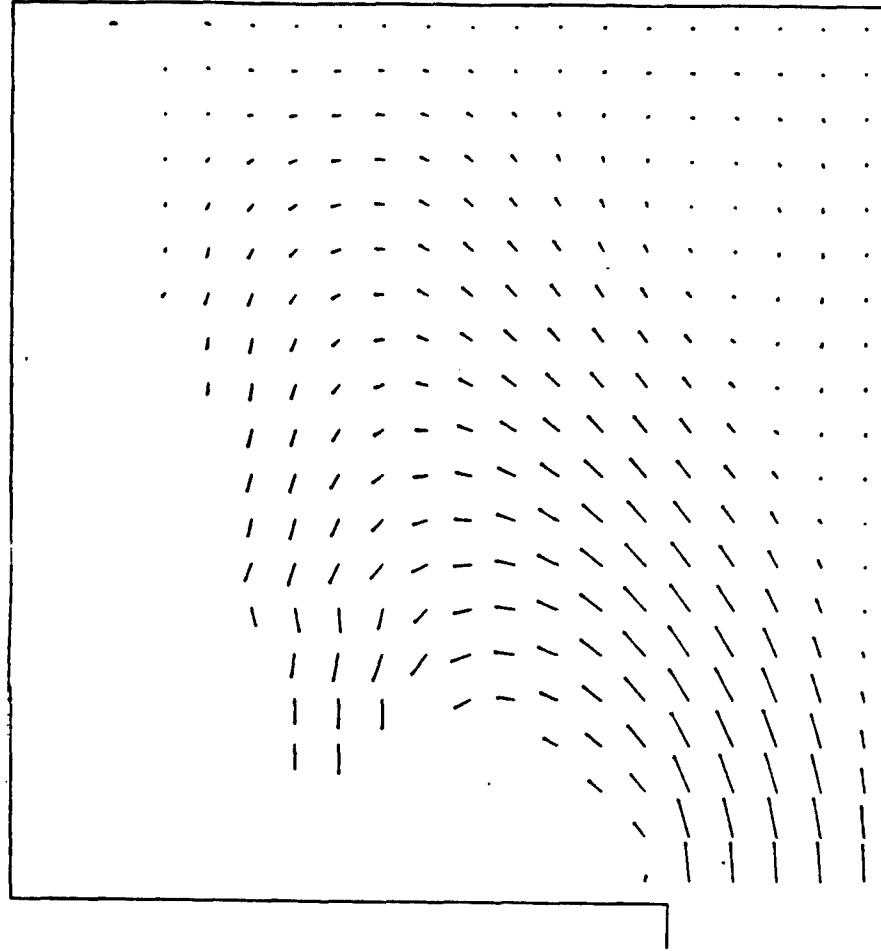


Figure V-5: Filling of a vertical plate, continued.

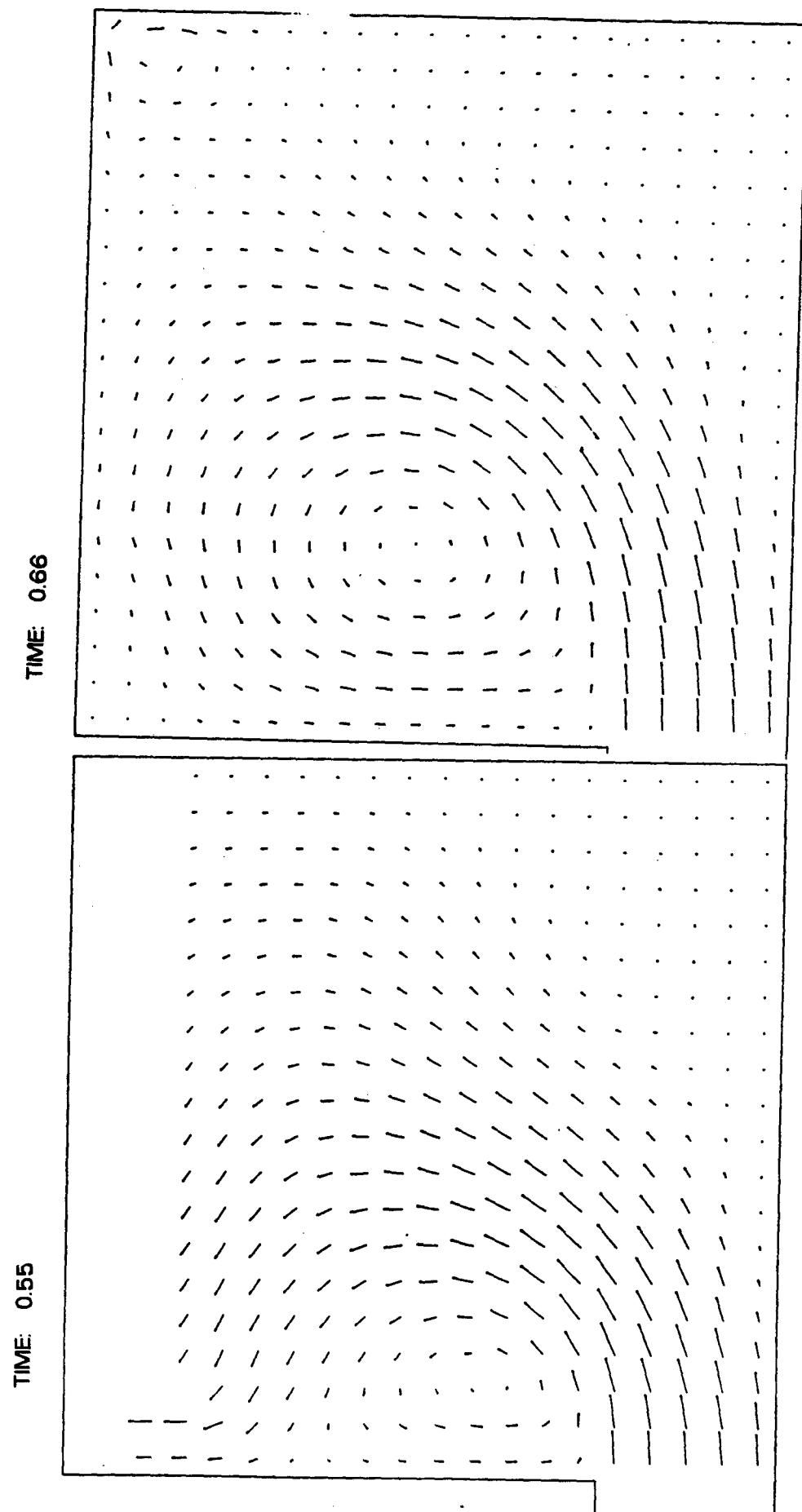
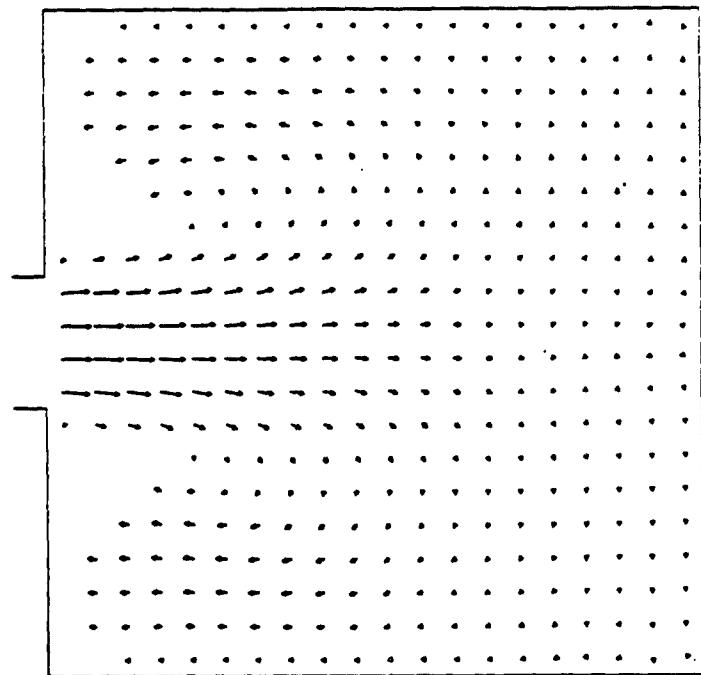
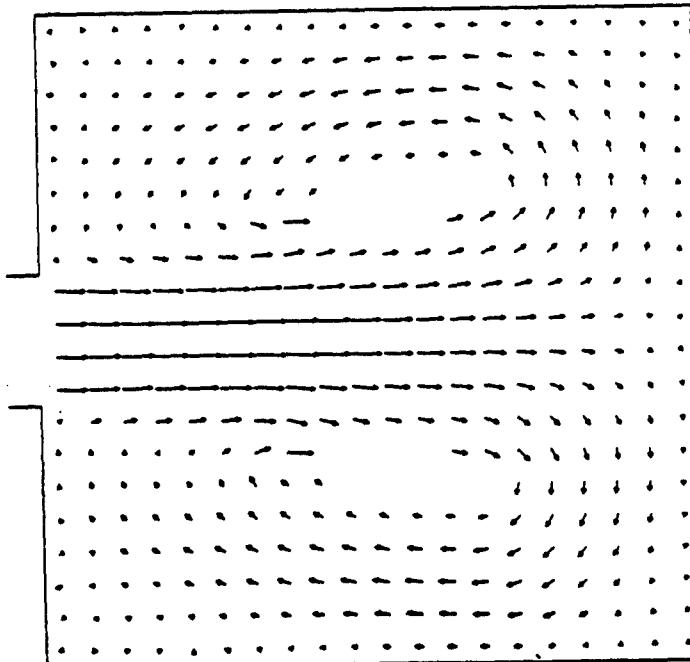


Figure V-5: Filling of a vertical plate, continued.



TIME: 9.50



TIME: 0.859

Figure V-6: Effect of filling rate on filling of a square mold cavity

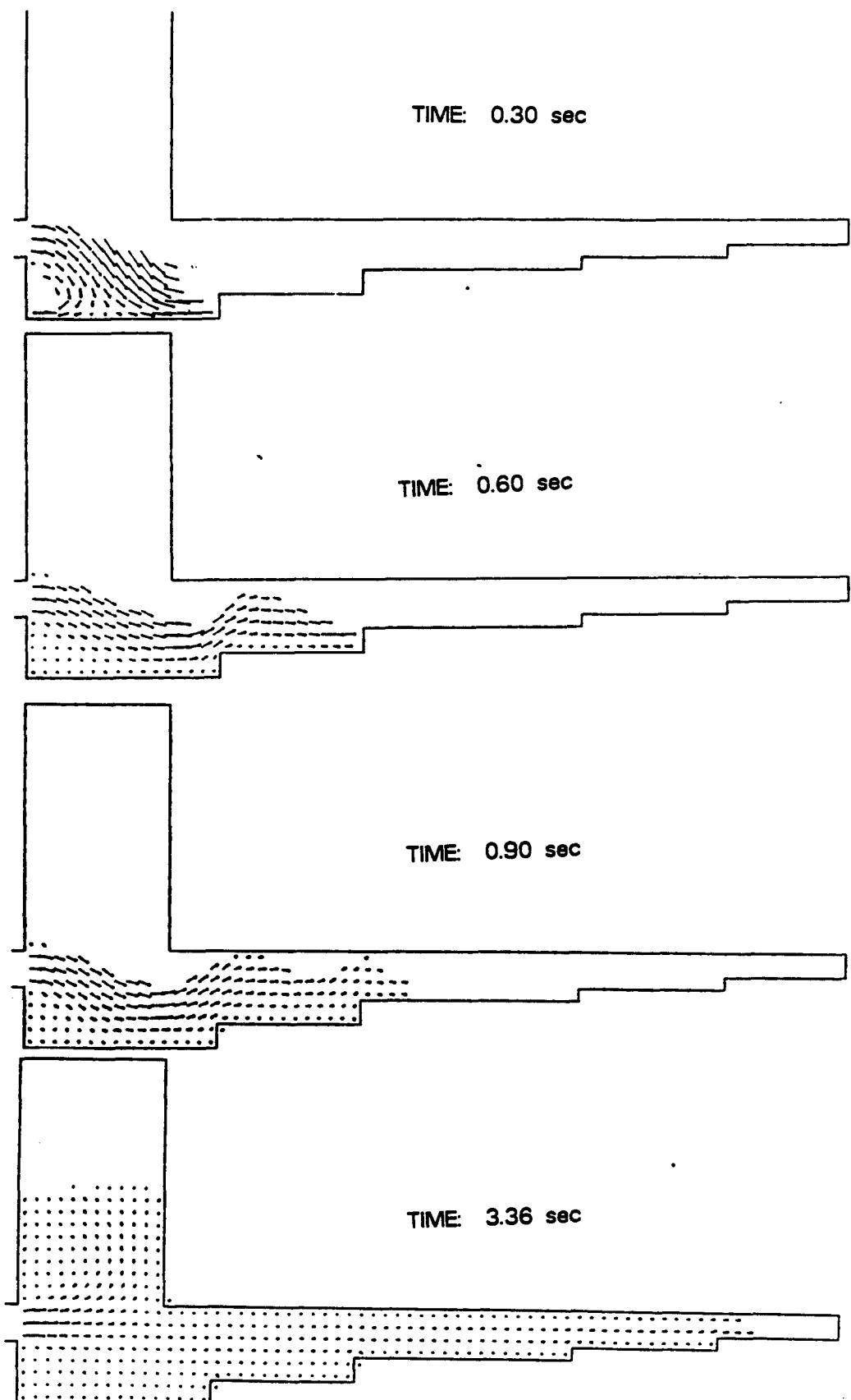


Figure V-7: Velocity vectors, simulated, for filling of plate A.

## V.4 INVESTMENT CASTING

The discussion of heat transfer analysis emphasized application to sand casting. The basic analyses also may be applied to permanent mold, die casting, and investment casting. However, the assumptions that shape the boundary conditions would have to be changed accordingly. Specifically, for investment casting:

1. The mold walls are sufficiently thin so that they may saturate with heat. Heat transfer to the ambient will be by convection (unless cast in vacuum) and by radiation.
2. Radiation and/or conduction from one section will cause heating of other regions within its "view".
3. The shell material may be translucent in the infrared and/or visible. Heat will pass through the shell by a combination of radiation and conduction.
4. Because section sizes generally are thin, it will be necessary to couple fluid flow and heat flow analyses.

## **V.5 SUMMARY**

Computer simulations have had limited use in design of foundry processes. The extent of application of rigorous models will expand as the cost of computing decreases, as the analysis routines are made more user friendly, and as the population becomes more computer literate. Heat flow analysis techniques are well developed and the heat transfer in castings is fairly well understood. Fluid flow analyses are now being developed and considerable experimentation to validate the models and expand our base of understanding is still required. The development of fluid flow models will allow an important step in modeling, namely coupling fluid flow and heat flow analyses.

## **ACKNOWLEDGEMENTS**

The author is indebted to Professor Robert A. Stoehr and Messrs. V. Srinath, W. S. Hwang, and P. Wisniewski of the University of Pittsburgh for making available information not yet fully reported in the literature. Support for work reported is under U.S. Army, Tank and Automotive Command contract DAAK-30-78-C-107, Mr. Michael Holly, Project Monitor.

## REFERENCES

1. W. S. Pellini, Trans. A.F.S., 61 (1953) pp. 61-80.
2. H. D. Brody et al., "Improved Foundry Castings Utilizing CAD/CAM"; Contract DAAK-30-78-C107, U.S. Army TACOM, Warren, MI., October 1981.
3. H. D. Brody and R. A. Stoehr, J. of Metals, 32 (Sept. 1980) pp. 20-27.
4. J. G. Henzel and J. Kevarian, J. of Metals, 17 (May, 1965), pp. 561-568.
5. R. D. Pehlke, M. J. Kirt, R. E. Marrone, and C. J. Cook, Cast Metals Research Journal, 9 (June, 1973) pp. 49-55.
6. J. Mathew and H. D. Brody, Solidification and Casting of Metals, The Metals Society (London, 1979) pp. 244-249.
7. M. C. Flemings, Solidification Processing, McGraw Hill (New York, 1972).
8. E. Niyama, T. Uchida, M. Morikawa, S. Saito, International Cast Metals Journal, 6 (June, 1981) pp. 16-22.
9. J. G. Henzel, Temperature Loss from Gating Systems, Heat Trans. Comm. - A.F.S. (Des Plaines, IL, 1966).
10. J. W. Hlinka, V. Paschkis, F. S. Puhr, A.F.S. Trans., 69 (1961) pp. 527-534.
11. J. E. Welch, F. H. Harlow, J. P. Shannon, and B. J. Dally, Tech. Report LA 3425, Los Alamos Scientific Laboratory (1966).
12. A. A. Amsden and F. H. Harlow, Tech. Report LA-4370, Los Alamos Scientific Laboratory (1970).

## VI. FLUID FLOW MODELING FOR CAD OF CASTINGS

W. S. Hwang and R. A. Stoehr

### SUMMARY

A comprehensive computer-aided-design system for producing castings must make use of several techniques to model the flow of molten metal through the sprue and runners and into the mold cavity. The techniques described in this report have proven to be useful, each in its own place. The techniques based on the Bernoulli and Saint-Venant equations are well established, but it is important to incorporate them in a computer system so that the mass of calculations required to obtain solutions, often iteratively, may be handled with ease, and so that the extensive data bases that are needed may be accessed efficiently. The marker-and-cell technique, although not previously used to any extent in metallurgical applications, appears to have a unique ability to model flow patterns within mold cavities. Taken together, these techniques can provide the designer with the information needed to improve casting quality and efficiency.

\* Originally published: W. S. Hwang and R. A. Stoehr, "Fluid Flow Modeling for Computer-Aided-Design of Castings," Journal of Metals, TMS-AIME, Oct. 1983, pp. 22-27.

## VI.1 INTRODUCTION

The development of computer-aided-design (CAD) systems for the production of castings is occurring at a rapid rate. The overall objective is to reduce the reliance on the art and experience of the individual designer and replace it with the systematic use of scientific principles and the accumulated experience of many designers. The University of Pittsburgh, Department of Metallurgical and Materials Engineering has been involved in the development of such a system for the U.S. Army Tank-Automotive Command.

Producing a casting involves many physical phenomena that occur simultaneously or in rapid succession. The relationships between these phenomena and how they affect the quality and cost of the product can be appreciated only if the CAD systems consider each of the factors involved. The importance of heat transfer and stress analysis is widely recognized as essential for a complete CAD system for designing castings. Fluid flow should also be recognized for its central role in the process and included in a similar way. Unfortunately, metallurgical engineers usually are less acquainted with fluid flow studies, and therefore require some encouragement to try to put them to practical use.

Objectives of fluid flow modeling for the computer-aided design of castings are:

- Insuring smooth flow during entry.
- Minimizing gas entrapment.
- Keeping dross and inclusions out of the mold cavity.
- Obtaining filling time within the desired range.
- Distributing the metal properly to the branches of a multiple gate system.
- Avoiding mold erosion.

Each of these items is commonly considered by the designer using experience and "rules of thumb". In this way, designs may be developed to include appropriate sprue taper; ratios between the cross-sections of gates, runners, and sprues; and the shape and placement of sprue and runner extensions to trap dross. For a CAD system, however, this must be placed on a more scientific basis, one that is amenable to calculation.

The following approaches to fluid flow modeling have been found to be useful for the design of castings, each for a different purpose:

- The Bernoulli equation approach - useful for flow of metal through full channels.
- The Saint-Venant equation approach - useful for flow of metal through partly filled runners.
- The marker-and-cell technique - useful for plotting the entry of metal with a free surface into the mold cavity.

## VI.2 USES OF THE BERNoulli EQUATION APPROACH

Four different uses of the Bernoulli or mechanical energy balance equation approach have been incorporated into the CAD system developed at the University of Pittsburgh. These are determination of mold filling time, design of sprue taper, design of chokes to equalize flow through multiple runners and gates, and determination of unequal rates of flow through multiple gates from a common runner.

A brief description of how each of these is accomplished will follow. The Bernoulli equation may be written as follows:\*

$$\frac{(P_2 - P_1)}{\rho} + \frac{V_2^2}{2b_2} + \frac{V_1^2}{2b_1} + g(Z_2 - Z_1) + E_f = 0 \quad (VI.1)$$

where

$b$  is a function of the Reynolds number

$E_f$  is a function of configuration, surface roughness,  $v$ , and  $V$ .<sup>1</sup>

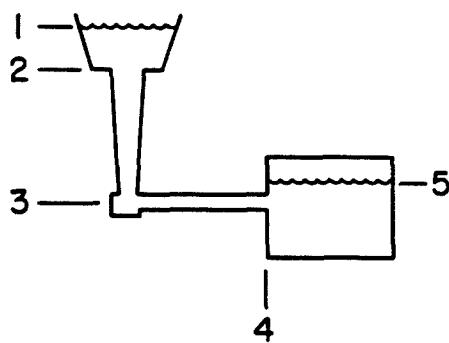
Figure VI-1 shows a system for which a filling time calculation is to be made. A list of the data needed to complete the calculation is shown on the figure. Calculation of the volume of the mold cavity as a function of level is important, and sometimes it is not easy if the casting shape is complex. Most CAD systems provide for the entry of a geometric description of the casting in digital form, usually as the first task. This provides the necessary data for the calculation of the volume versus level information. Similarly the dimensions of all the channels may be entered in digital form.

---

\*See Table of Symbols for definition of terms.

## **DETERMINATION OF FILLING TIME**

---



### **DATA NEEDED:**

- Dimensions of all channels
- Level of metal in pouring basin
- Volume of mold cavity as a function of level
- Friction factors and friction loss coefficients
- Viscosity and density of molten metal

**Figure VI-1:** Factors in the determination of the filling time of a mold.

The viscosity and density of pure metals near their melting temperatures are usually well known. Most castings, however, are made from alloys for which these data may not be as reliable. The importance of building this data base will be more obvious as CAD systems routinely make use of this information.

The friction factors and friction loss coefficients that go into the calculation of the friction energy loss term  $E_f$  in this equation are reliably known for straight pipes and standard junctures such as sharp elbows. References back to Moody may be utilized.<sup>1-3</sup> In a mold, however, connections between channels are seldom standard or simple. The sprue, for example, may be enlarged and extended where it meets the runner. Therefore, additional tabulations of data are needed to cover these. Users of CAD systems may have to determine some of these factors experimentally to add to their own data bases.

Filling time calculations are based on assumptions about the level of metal in the pouring basin. Furthermore, they are based on the assumption that the channels will be full of molten metal from the pouring basin to some recognized choke point. Ideally this choke point will be at the gate. Sometimes, however, it may be where the pouring basin connects to the sprue, resulting in free fall of the molten metal through the sprue. The Bernoulli equation can be used to predict this and can be used to derive equations for the proper tapering of the sprue to avoid this problem, as shown in Figure VI-2.<sup>4</sup> This may be reduced to a simple equation for the ideal frictionless case with the velocity at the top surface of the metal equal to zero and with no buildup of back pressure in the system. This equation is

$$\frac{A_3}{A_2} = \frac{[Z_1 - Z_2]^{1/2}}{[Z_1 - Z_3]^{1/2}} \quad \text{or} \quad \frac{D_3}{D_2} = \frac{[Z_1 - Z_2]^{1/4}}{[Z_1 - Z_3]^{1/4}} \quad (\text{VI.2})$$

where

$A_2$  and  $A_3$  are the cross-sectional areas at top and bottom of the sprue

$Z_1$ ,  $Z_2$ , and  $Z_3$  are the elevations at planes 1, 2, and 3, respectively.

If friction is considered, this equation should be modified to

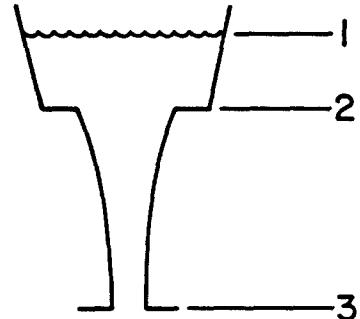
$$\frac{A_3}{A_2} = \frac{[Z_1 - Z_2]^{1/2}}{[Z_1 - Z_3 - (Z_1 - Z_2)[e_{f2,3} + (Z_2 - Z_3)[e_{f1,2}]]^{1/2}} \quad (\text{VI.3})$$

The friction energy loss coefficients  $e_{f2,3}$  and  $e_{f1,2}$  are always positive, and thus it may be seen that friction within the sprue  $e_{f2,3}$  decreases the required taper while friction above the sprue increases it.

## SPRUE TAPER DESIGN

1. For ideal frictionless case with  $\bar{V}_1 = 0$  and  $P_1 = P_2 = P_3$

$$\frac{A_3}{A_2} = \sqrt{\frac{z_1 - z_2}{z_1 - z_3}}$$



2. For case where friction is considered

$$\frac{A_3}{A_2} = \sqrt{\frac{z_1 - z_2}{z_1 - z_3 - (z_1 - z_2) \sum e_{f,2,3} + (z_2 - z_3) e_{f,1,2}}}$$

Figure VI-2: Sprue taper design to avoid free fall turbulence.

At times it is desirable to feed two mold cavities simultaneously from a common sprue or runner. In the case shown in Figure VI-3 two runners of unequal length are being used to feed two cavities from a common sprue. In order to provide equal filling rates for both cavities it is necessary to place a choke as shown in the shorter runner to equalize friction energy losses in both. Such chokes are assigned an equivalent length-to-diameter ratio and when used with runners of *equal* cross-sectional area, the following equations apply.

$$\begin{aligned} A_1 &= A_2 & V_1 &= V_2 & f_1 &= f_2 \\ D_1 &= D_2 & Q_1 &= Q_2 & \beta_1 &= \beta_2 \\ (L_2/D_2) &= (L_2/D_2) + (L/D)_{\text{equivalent}} \end{aligned} \quad (\text{VI.4})$$

If the runners are of unequal area the following equations apply.

$$V_1 \text{ unequal } V_2 \text{ but } A_2 V_2 \text{ equal } A_3 V_3$$

$$\begin{aligned} 1/b_1 + e_{f1} + 4f_1(L/D)_1 &= \\ (A_1/A_2)^2 \{ 1/b_2 + e_{f2} + 4f_2(L_2/D_2) + 4f_2(L/D)_{\text{equivalent}} \} \end{aligned} \quad (\text{VI.5})$$

Indeed it may be seen that this equation may be satisfied even if  $(L/D)_{\text{equivalent}}$  is zero, if the proper ratios of  $(L_1/D_1)$  and  $(L_2/D_2)$  are chosen. This would be simple except for the fact that  $b$ ,  $e_f$ , and  $f$  are all functions of the Reynolds number, which in turn depends on  $V$ . Thus, an iterative method of solution is required.

The problem of determining *unequal* flow rates through multiple gates may also be approached through the Bernoulli equation.<sup>5</sup> A situation involving four such gates is shown in Figure VI-4. To analyze it, the section between the plane marked Station A and Gates 1 and 2 may be represented by the following equations.

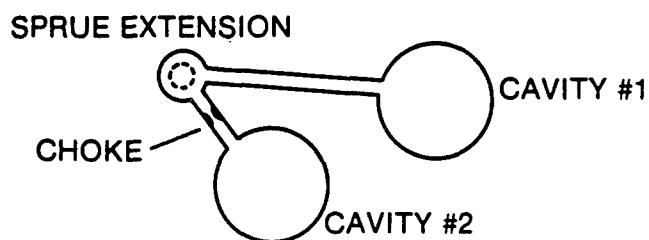
$$\frac{V'^2}{2} + \frac{P_A}{\rho} = \frac{V_1^2}{2} + \frac{P_0}{\rho} + \alpha \frac{V_1^2}{2} \quad (\text{VI.6})$$

$$\frac{V'^2}{2} + \frac{P_A}{\rho} = \frac{V_2^2}{2} + \frac{P_0}{\rho} + \beta \frac{V_2^2}{2} \quad (\text{VI.7})$$

$$V_1^2 + \alpha V_1^2 = V_2^2 + \beta V_2^2 \quad (\text{VI.8})$$

## **CHOKE DESIGN FOR EQUAL FLOW THROUGH MULTIPLE RUNNERS OR GATES**

---



For runners of equal area

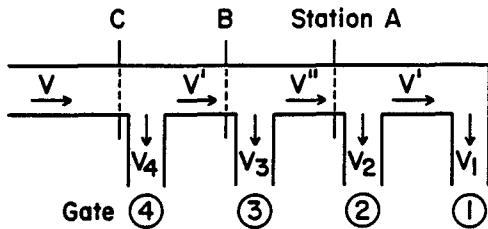
$$A_1 = A_2 \quad V_1 = V_2 \quad Q_1 = Q_2$$

$$D_1 = D_2 \quad f_1 = f_2 \quad \beta_1 = \beta_2$$

$$\frac{L_1}{D_1} = \frac{L_2}{D_2} + \left( \frac{L}{D} \right) \text{ equivalent for choke}$$

**Figure VI-3:** Equal flow through multiple runners from a common sprue.

**DETERMINING UNEQUAL FLOW RATES  
THROUGH MULTIPLE GATES**



Bernoulli eqn. between station A and gate ①  
and between station A and gate ②

$$\frac{V''^2}{2g} + \frac{P_A}{w} = \frac{V_1^2}{2g} + \frac{P_0}{w} + \alpha \frac{V_1^2}{2g}$$

$$\frac{V''^2}{2g} + \frac{P_A}{w} = \frac{V_2^2}{2g} + \frac{P_0}{w} + \beta \frac{V_2^2}{2g}$$

$$\frac{V_1^2}{2g} + \alpha \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + \beta \frac{V_2^2}{2g}$$

**Figure VI-4:** Unequal flow through multiple gates from a single runner.

which gives

$$\frac{V_1}{V_2} = \frac{[1 + \beta]^{1/2}}{[1 + \alpha]^{1/2}} \quad \text{or} \quad \frac{Q_1}{Q_2} = \frac{A_1}{A_2} \cdot \frac{[1 + \beta]^{1/2}}{[1 + \alpha]^{1/2}} \quad (\text{VI.9})$$

The evaluation of  $\alpha$  and  $\beta$  is the essential step in the use of these equations.

To evaluate  $\alpha$  note that the frictional energy loss between Station A and Gate 1 is composed of:

1. Loss in flowing past the tee at entrance to Gate 2.
2. Loss in the runner between Tee 2 and the end of the runner.
3. Bend loss at the end of the runner.
4. Loss due to contraction entering Gate 1.

For example:

$$\alpha V_1^2 = C_1 V^2 + f_1 (L/D') V^2 + C_2 V^2 + C_3 V_1^2 \quad (\text{VI.10})$$

The friction factor  $f_1$  may be evaluated from the work of Moody,<sup>2,3</sup> etc., and for a typical case may turn out to be 0.02. The factors for flow past the tee, for bend loss at the end of the runner, and for loss due to the contraction entering the gate are based on the work of Ruddle<sup>5</sup> and of Giesecke and Badgett,<sup>6</sup> and they are functions of flow rates in different branches of the runner. The values for the coefficients  $C_1$ ,  $C_2$ , and  $C_3$  must be found iteratively. For the present problem they turn out to be 0.58, 1.8, and 0.33, respectively, and this yields a value of  $\alpha = 0.48$ . Then to evaluate  $\beta$  note that the frictional energy loss between Station A and Gate 2 is composed of loss flowing around Tee 2 and contraction loss between runner and Gate 1. This gives rise to the following equation:

$$\beta V_2^2 = C_4 V_2^2 + C_3 V_2^2 \quad (\text{VI.11})$$

where  $C_4$  and  $C_3$  are friction energy loss coefficients which must be found iteratively. For this example they are 4.1 and 0.33, respectively, which gives  $\beta = 4.43$  and the following results:

$$\frac{V_1}{V_2} = \frac{[1 + \beta]^{1/2}}{[1 + \alpha]^{1/2}} = \frac{[1 + 4.43]^{1/2}}{[1 + 0.48]^{1/2}} = 1.915 \quad (VI.12)$$

and

$$Q_1/Q_2 = 1.915$$

This procedure is then carried back up stream to Station B between Gates 3 and 4 and later to Station C. The results of the calculation for this example is shown in Figure VI-5. The greatest flow is through Gate 1, followed by that through Gates 4, 3, and 2 in order.

Although the principles used in this application are old, the amount of trial and error needed to fit the friction loss factors to the flow rates requires the use of a modern computer to do the calculations in a practical amount of time.

### VI.3 THE SAINT-VENANT EQUATION APPROACH

A different approach is required to model flow of metal in partly filled runners and to determine whether runners are pressurized. This requires the use of the Saint-Venant equations.<sup>7</sup> These are also based on the energy balance principle, but are designed to handle the situation where the bottom of the channel and the free top surface of the fluid slope, each at different rates. Furthermore, they may be written to handle transient situations. They have been used extensively in civil engineering to calculate flows through river valleys, irrigation ditches, and similar places, including those following rupture of dams.

As shown in Figure VI-6, the bottom slope of the channel is referred to the coordinate Z relative to an imaginary horizontal plane. The coordinate Y refers to the depth of the fluid in the channel. The appropriate equation of continuity is

$$W_T \frac{\partial Y}{\partial t} + \frac{\partial (VA)}{\partial X} = 0 \quad (VI.13)$$

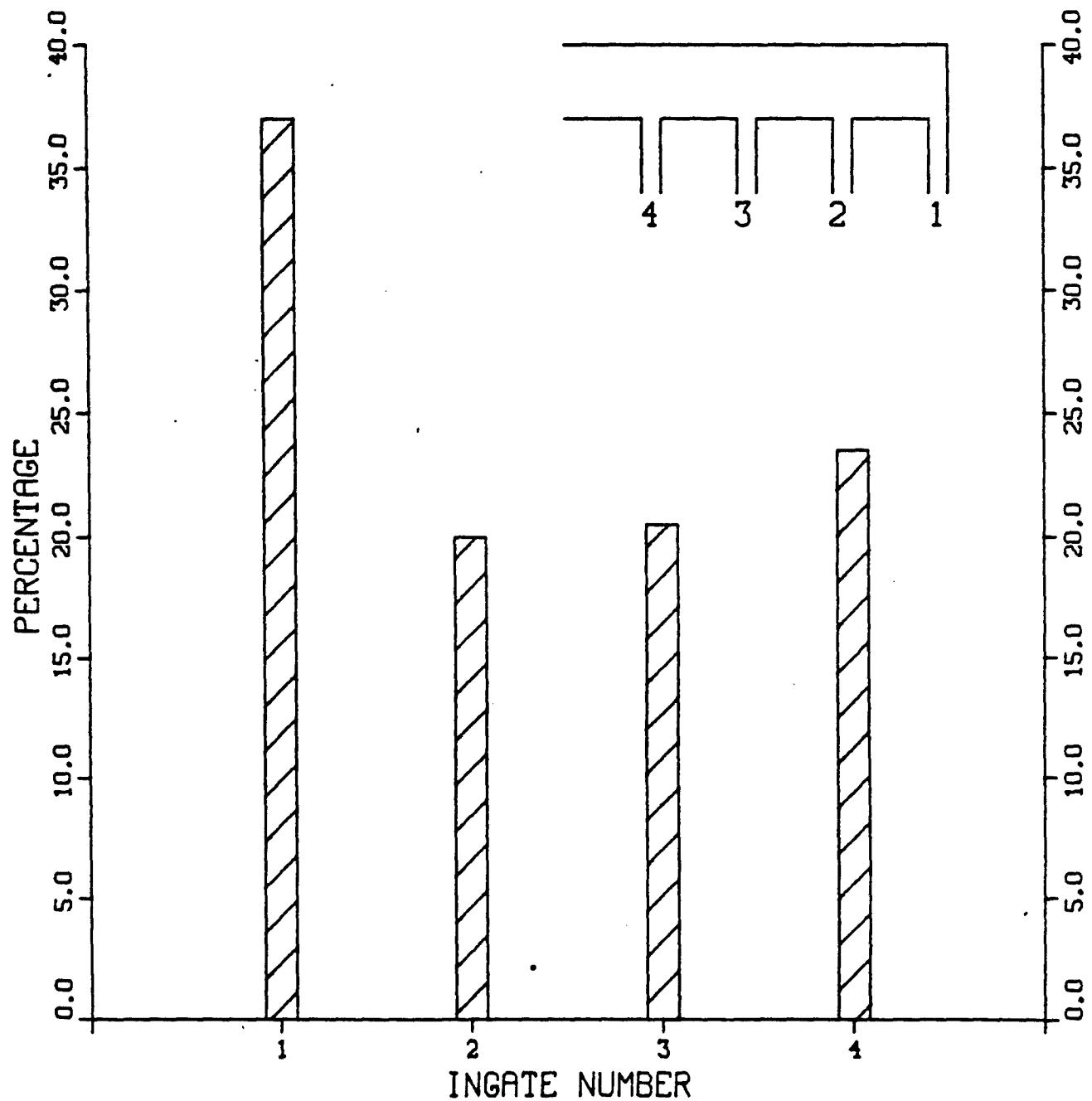
where

$W_T$  is the width of the channel at fluid depth Y.

VA is the product of flow velocity and fluid area, i.e., the volumetric flow rate of fluid at a particular value of X.

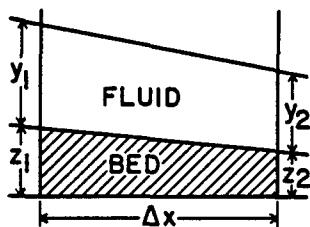
X coordinate is measured along the channel and follows any

## DISTRIBUTION FOR MULTIPLE INGATE



**Figure VI-5:** Flow distribution in gates shown in Figure VI-4.

## THE SAINT-VENANT EQNS.



Bottom slope

$$S_0 = \frac{z_1 - z_2}{\Delta x}$$

Friction slope

$$S_f = \frac{n^2 V^2}{R^{4/3}}$$

where

$R$  = hydraulic radius

$n$  = Manning's roughness coefficient

**CONTINUITY EQN.**

$$w_T \frac{\partial y}{\partial t} + \frac{\partial (VA)}{\partial x} = 0$$

**EQN. OF MOTION**

$$\frac{V}{g} \frac{\partial V}{\partial x} + \frac{\partial y}{\partial x} - S_0 + S_f = 0$$

or

$$d(z + y + \frac{aV^2}{2g}) = -S_f dx - \frac{1}{g} \frac{\partial V}{\partial t} dx$$

Figure VI-6: Saint-Venant representation of flow along a sloping runner.

meandering that may exist

Essentially this equation says that the depth of the liquid at any  $X$  changes in response to the variation in  $VA$  along the channel.

For the steady-state case, the energy balance equation can be written

$$\frac{V}{g} \frac{\partial V}{\partial X} + \frac{\partial Y}{\partial X} - S_o + S_f = 0 \quad (VI.14)$$

In this equation the first term represents the kinetic energy change, the combination of the second and third terms represents the change in the potential energy, and the fourth term  $S_f$  (the friction slope) represents the friction energy loss.

$S_f$  is given by

$$S_f = \frac{n^2 V^2}{R_h^{4/3}} \quad (VI.15)$$

where

$R_h$  = hydraulic radius

$n$  = Manning's roughness coefficient

For the time-dependent case, the energy balance equations becomes

$$\frac{V^2}{2g} + \frac{1}{g} \frac{\partial V}{\partial t} - S_f \frac{dX}{dt} = - \frac{d}{dt} \left( Z + Y \right) \quad (VI.16)$$

in which the last term on the right represents the acceleration.

Note that roughness factor  $n$  takes into account the channel configuration as well as surface roughness and it may change frequently along the course of the channel. It is larger at the tip of the entering stream than in areas where a layer of molten metal is already present. Nevertheless, the Saint-Venant equations represent the simplest approach to flow of liquid metal in partly filled runners.

## VI.4 THE MARKER-AND-CELL TECHNIQUE

The marker-and-cell (MAC) technique represents the only known method for determining flow patterns during the filling of mold cavities.<sup>8,9</sup> Here flow is transient and has a free surface, but unlike those situations that may be handled by the Saint-Venant equations, flow is not confined to follow a certain channel.

Instead of using the energy balance principle, the MAC technique uses the momentum balance principles embodied in the Navier-Stokes equations. For two-dimensional cartesian coordinates, in the form best suited for conversion to the finite difference form, these equations are:

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} = - \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial x^2} + v \frac{\partial^2 u}{\partial y^2} + g_x \quad (\text{VI.17})$$

$$\frac{\partial v}{\partial t} + \frac{\partial v^2}{\partial y} + \frac{\partial uv}{\partial x} = - \frac{\partial p}{\partial y} + v \frac{\partial^2 v}{\partial x^2} + v \frac{\partial^2 v}{\partial y^2} + g_y \quad (\text{VI.18})$$

The equation of continuity in this case is:

$$D = (\partial u / \partial x) + (\partial v / \partial y) = 0 \quad (\text{VI.19})$$

The equations are written in finite difference form in both time and space. The space representing the mold cavity is divided into a fixed (or Eulerian) mesh of "cells." As incoming molten metal enters this space, imaginary "markers" are pictured as entering with it. An additional set of markers enters with each new time step. The markers move with the fluid. The movement of the markers is calculated at each time step, thus defining the location of the molten metal. The marker locations are used to designate each cell as being *full*, partly full or *surface*, or *empty*.

The boundary conditions at the free surface may be stated as follows:

1. Stress tangential to the surface vanishes.
2. Stress normal to the surface exactly balances the externally applied normal stress. (This may be set to zero if the external pressure is uniform on all surfaces.)

The boundary conditions at walls may be set up to represent free-slip or no-slip conditions.

A block flow chart for this program is shown in Figure VI-7. The  $D_{ij}$  referred to is the finite difference form of the divergence for cell  $ij$ . Although the divergence is postulated to be zero for all full cells, the value of  $D_{ij}$  may not necessarily be zero due to the finite difference representation of this function. The  $R_{ij}$  referred to is a factor based on the present values of the velocity components and their derivatives. It is used to calculate the values of the pressures  $p_{ij}$  by a Poisson type equation:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = -R \quad (VI.20)$$

$R_{ij}$  is derived so as to make the divergence equal zero again at the next time step. Note that the values of  $p$  are calculated to maintain agreement with the continuity equation, rather than being measured. The calculated values of  $p$  are then used in the finite difference form of Equations (VI.17) and (VI.18) to calculate the velocity vectors at the next time step. This calculation cycle is repreated for as many times steps as desired.

The viscosity used in these calculations must be several orders of magnitude higher than the material viscosity for two reasons.<sup>10</sup> First, the finite difference equations do not have stable solutions unless the viscosity has certain minimum values. Second, flow of molten metals into mold cavities is usually turbulent and so the effective viscosity, which is the sum of the material and turbulent viscosities, is much higher than the material viscosity alone. The turbulent viscosity is a function of flow conditions existing within the system and varies with time and position. Although it would be very desirable to use a model for the effective viscosity that would follow these variations, this has proved impractical in view of the large increase in computer time and core that it would require. Therefore, a constant value of  $\nu$  is used which satisfies the stability requirements and approximates the effective viscosity.

Some of the results obtained by the use of this method are shown in Figures VI-8 and VI-9. Figure VI-8 shows the molten metal entering a horizontal rectangular cavity through a gate in the left side. The stream expands a little bit before hitting the opposite wall. When it hits the opposite wall, it divides symmetrically into two streams which flow outward along this wall, then backward along the two side walls. Two spots, one on each side of the ingate, are the last to fill. Small vortices form in this region. With a larger flow rate (11 times larger than the first), the metal enters the mold like a jet. It hits the opposite wall and flows outward along it in two symmetrical streams that cling much closer to the wall than in the slower moving case. These streams race backward against the two

## Flow chart of FPMAC program

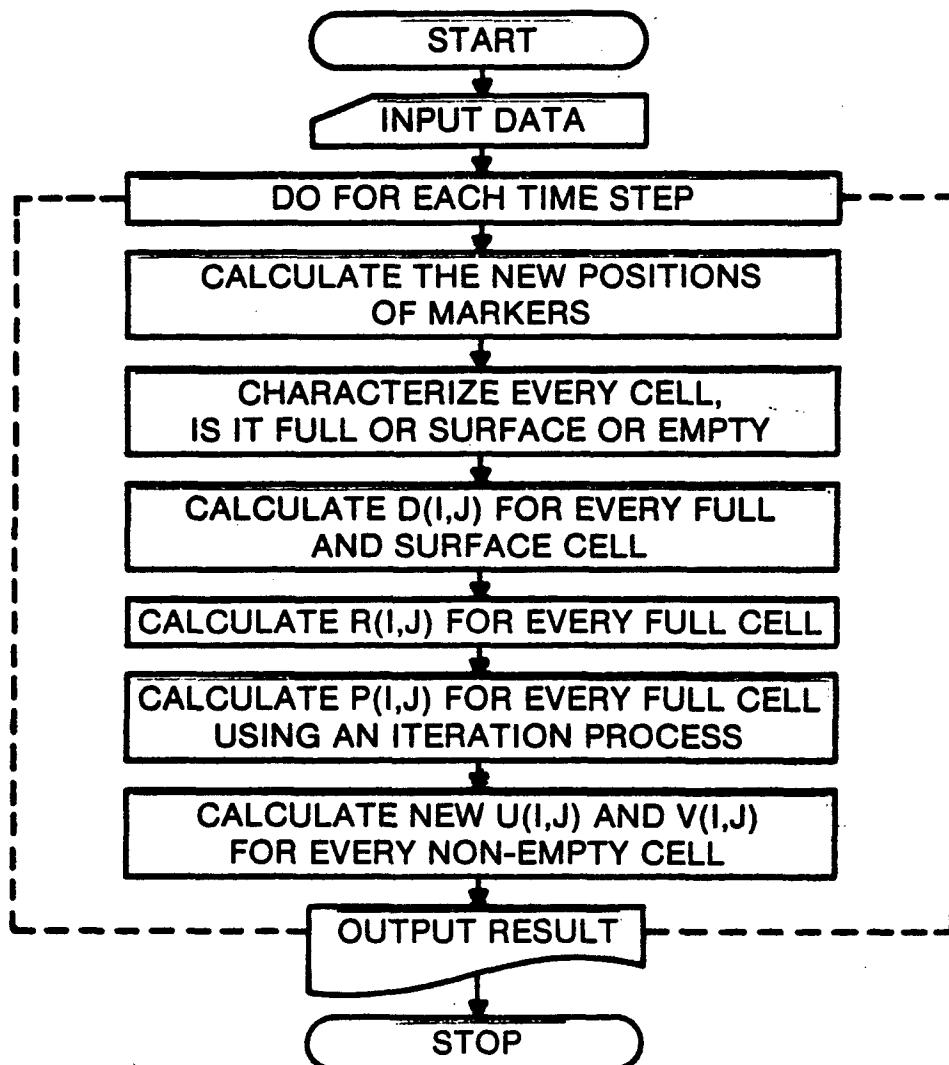
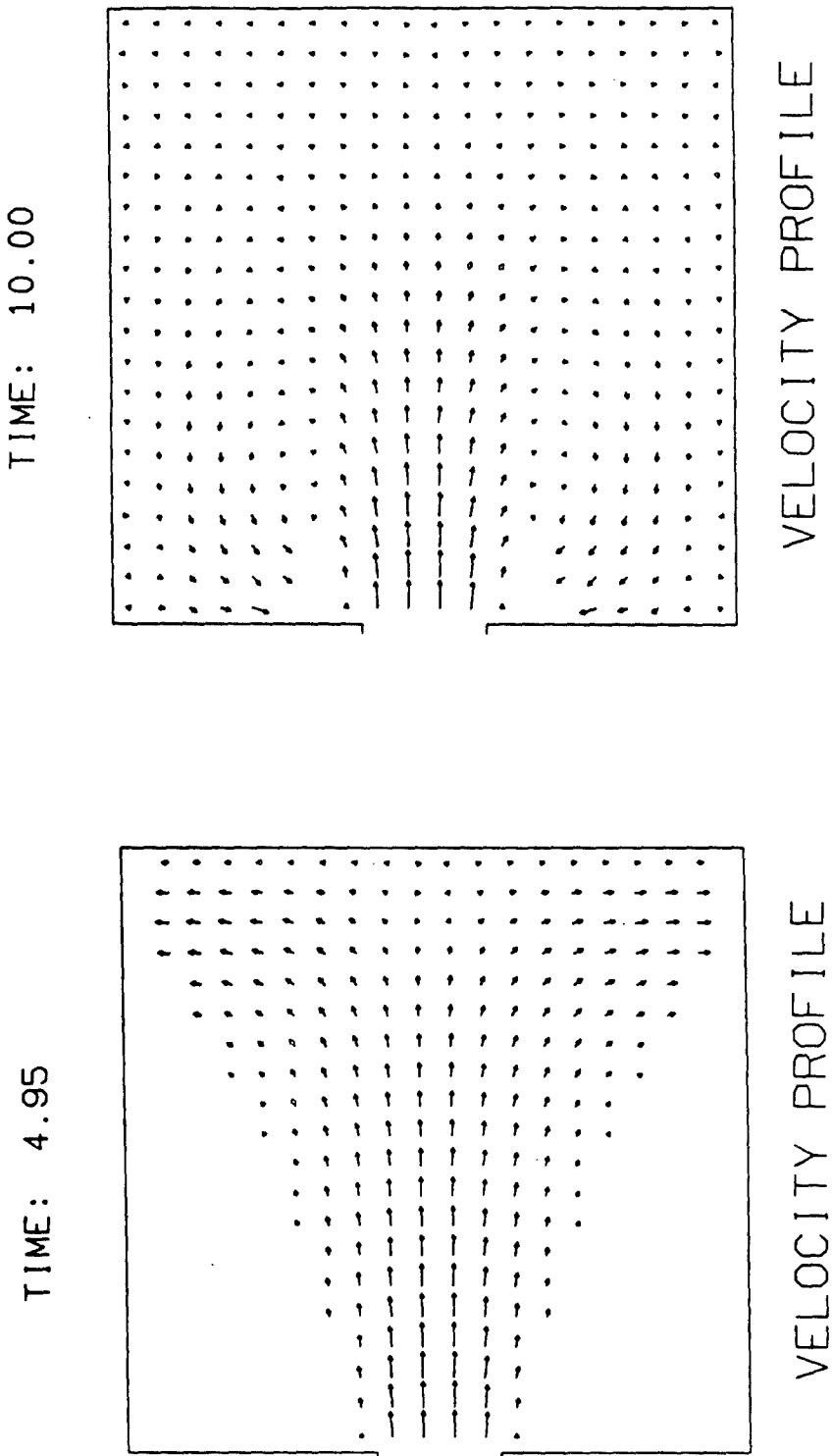
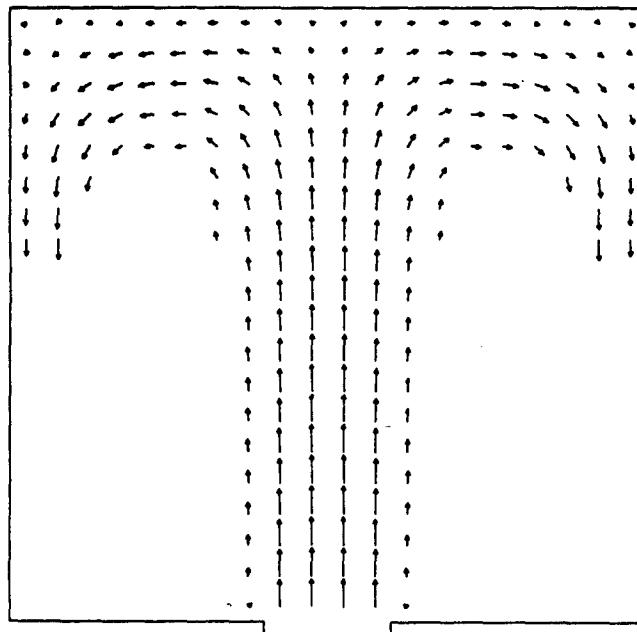


Figure VI-7: Block flow chart of the MAC calculation scheme.

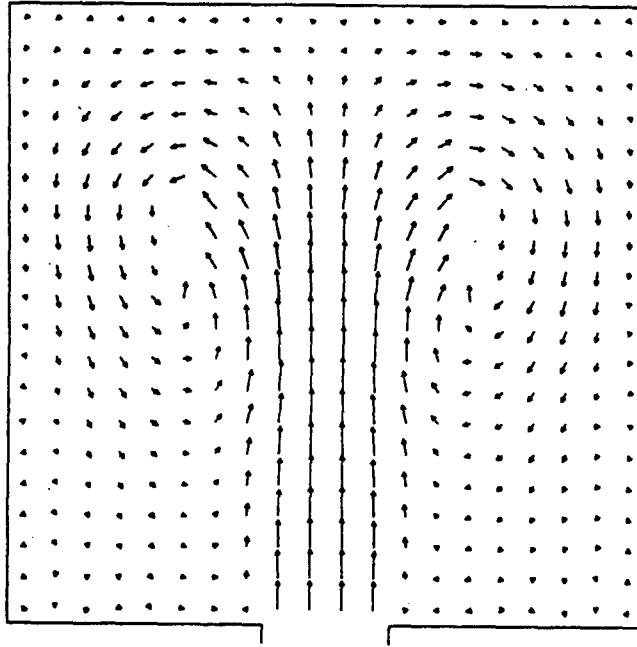


**Figure VI-8:** Vector plots (MAC) of flow into horizontal square molds.

TIME: 0.459



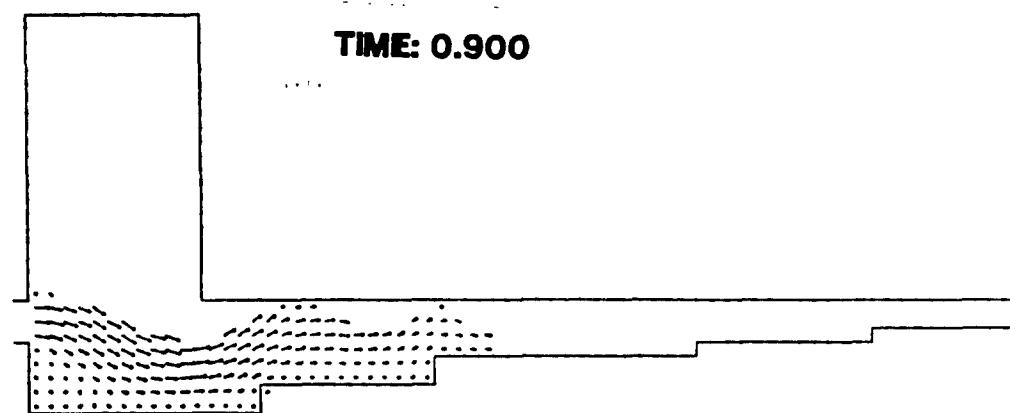
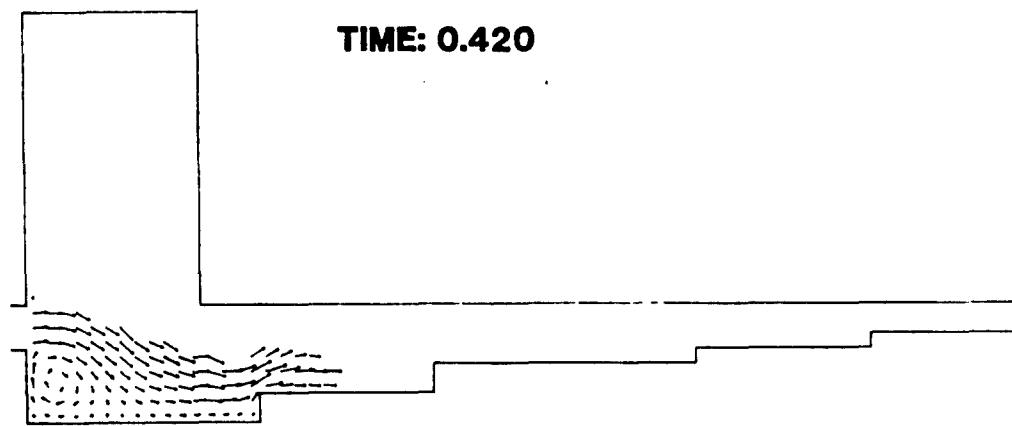
TIME: 0.899



VELOCITY PROFILE

VELOCITY PROFILE

Figure VI-8: Vector plots of flow into square molds, continued



**Figure VI-9:** Vector plots of flow into mold with vertical steps on drag.

side walls, then along the ingate wall where they encounter the incoming jet. The last areas to fill are farther away from the ingate wall than in the first case, and two large vortices develop, one on each side of the centerline. The flow conditions in this example are similar to those experienced in die casting, and show how pockets of gas become entrapped in the incoming metal. Physical verification of these flow patterns has been obtained from high-speed motion pictures of the entry of water into transparent plastic models and from x-ray films of molten metal entering molds.<sup>11,12</sup>

A more complicated situation is shown in Figure VI-9 where molten metal enters a mold cavity containing steps. Vector plots are shown at two times. They show how standing waves develop at the steps, and how metal touches the top of the cavity above the first step before any reaches the far end of the cavity. They also show a strong vortex under the entry point at the earlier time, that weakens later as the mold fills.

The ability to show where vortices and dead zones develop is one of the most powerful things about this technique, since this information is not readily obtained by physical experiments.

#### ACKNOWLEDGMENTS

These fluid flow modeling techniques have been developed as part of a computer-aided-design system for improved foundry casting under Contract No. DAAK-30-70-C-108 from the U.S. Army Tank-Automotive Command, Research and Development Center, Warren, Michigan. Mr. Michael Holly, project monitor. Professor Harold D. Brody, University of Pittsburgh, has been the project director.

**TABLE OF SYMBOLS**

<u>Symbol</u>	<u>Equations</u>	<u>Meanings</u>
<u>upper case</u>		
A	13	Cross-section area of fluid perpendicular to flow direction ( $m^2$ )
$A_i$	2,3,4,5,9	Cross-section area ( $m^2$ )
$C_i$	10,11	Friction energy loss coefficients for specific sections of multiple gate system (dimensionless)
D	19	Divergence ( $s^{-1}$ )
$D'$	10	Diameter of runner between Gate 2 and entry to Gate 1 (m)
$D_i$	2,4,5	Diameter of circular cross section (m)
$E_f$	1	Friction energy loss (joules/kg)
$L'$	10	Length of runner between Gate 2 and entry to Gate 1 (m)
$L_i$	4,5	Length of runner or other channel (m)
$(L/D)_{\text{equivalent}}$	4,5	Equivalent length/diameter ratio to account for elbows, etc. (dimensionless)
$P_A$	6,7	Pressure at Station A ( $N/m^2$ )
$P_i$	1	Pressure ( $N/m^2$ )
$P_o$	6,7	Pressure inside mold cavity ( $N/m^2$ )
$Q_i$	4,9,12	Volumetric flow rate ( $m^3/s$ )
R	20	An algebraic function of the present values of the velocity components and their derivatives ( $s^{-1}$ )
$R_h$	15	Hydraulic radius (m)
$S_f$	14,15,16	"Friction slope," head equivalent to friction energy loss per unit length of channel (m/m)
$S_o$	14	Slope of channel bottom (m/m)
V	13,14,15,16	Flow velocity (m/s)
$V',V'',V'''$	6,7,10	Velocity through runner leading to specific gates ( $m/s^2$ ) Fig VI-4
$V_i$	1,4,5,6,7,8,9	Average velocity in runner or gate (m/s)
$W_T$	13	Width of channel at the top surface of the fluid (m)
X	13,14,16	Distance measured along channel in the

		direction of flow (m)
Y	13,14,16	Depth of fluid in channel (m)
Z	16	Elevation of bottom of channel (m)
$Z_i$	1,2,3	Elevation (m)
<u>lower case</u>		
b	1,4,5	Velocity distribution factor: 0.5g for laminar flow with parabolic distribution, 1.0 for plug flow
$e_{fi}$	3,5	Friction energy loss coefficient for contractions, expansions, changes of direction, etc. (dimensionless)
$f_i$	4,5,10	Friction factor (dimensionless)
g	1	Acceleration of gravity (9.8 m/s <sup>2</sup> )
$g_x$	17	Component of gravity (m/s <sup>2</sup> )
$g_y$	18	Component of gravity (m/s <sup>2</sup> )
n	15	Manning's roughness coefficient (s <sup>2</sup> /m <sup>2/3</sup> )
p	17,18,20	Pressure/density (m <sup>2</sup> /s <sup>2</sup> )
t	16,17,18	Time(s)
u	17,18,19	Component of velocity (m/s)
x	17,18,19,20	Spatial coordinate in cartesian system (m)
y	17,18,19,20	Spatial coordinate in cartesian system (m)
<u>Greek</u>		
$\alpha$	6,8,9,10	Friction energy loss between Station A and Gate 1 (dimensionless)
$\beta$	6,8,9,10	Friction energy loss between Station A and Gate 2 (dimensionless)
$\nu$	1,17,18	Kinematic viscosity (m <sup>2</sup> /s)
$\rho$	1,6,7	Density (kg/m <sup>3</sup> )

## REFERENCES

1. G.H. Geiger and D.R. Poirier, Transport Phenomena in Metallurgy, Chapters 3-4, Addison-Wesley, New York, New York, 1973.
2. L.F. Moody, "Friction Factors for Pipe Flow," Transactions of ASME, 66 (1944), pp. 671-684.
3. L.F. Moody, "An Approximate Formula for Pipe Friction Factors," Mechanical Engineering, 66 (1947), pp. 1005-1006.

4. S.I. Karsay, Ductile Iron III, Gating and Risering, Fer et Titans Inc., Sorel, Quebec, 1980.
5. R.W. Ruddle, The Running and Gating of Sand Castings, The Institute of Metals, Grosvenor Gardens, London, 1958, pp. 165-172.
6. F.E. Giesecke and W.H. Badgett, "Supplementary Friction Heads in One-Inch Cast Iron Tees," Heating, Piping, and Air Conditioning 4 (1932), pp. 53-56.
7. D.H. St John, K.G. Davis, and J.G. Magny, "Computer Modelling and Testing of Metal Flow in Gating Systems," CANMET, Energy, Mines and Resources Canada: Internal Report MRP/PMRL 80-12(J), 1980.
8. F.H. Harlow and J.E. Welch, "Numerical Calculations of Time-Dependent, Viscous, Incompressible Flow of Fluids with Free Surface," The Physics of Fluids, 8(12) (1965), pp. 2182-2189.
9. J.E. Welch, F.H. Harlow, J.P. Shannon, and B.T. Dally, "The MAC Method - A Computing Technique for Solving Viscous, Incompressible, Transient Fluid-Flow Problems Involving Free Surfaces," Los Alamos Scientific Laboratory Report, LA-3425, 1965.
10. C.W. Hirt, "Heuristic Stability Theory for Finite Differences Equations," Journal of Computational Physics, 2(1970), pp. 339-355.
11. "Water Analogy Studies - Flow and Gating of Castings," film produced by Case Institute of Technology for American Foundrymen's Society Training and Research Institute, Des Plaines, Illinois, and Die Casting Research Foundation, Inc., New York, New York.
12. M.C. Ashton and R.K. Buhr, "Direct Observation of the Flow of Molten Steel in Sand Molds," CANMET, Energy, Mines and Resources Canada: Internal Report PM-M-73-5, 1973.

## VII. METAL FLOW WITH FREE SURFACE INTO MOLDS.

R. A. Stoehr and W. S. Hwang  
University of Pittsburgh  
Pittsburgh, PA 15261

### SUMMARY

Modeling the transient flow of molten metal having a free surface requires techniques not previously used to any extent in metallurgical engineering. The Marker-and-Cell (MAC) technique and its simplified derivative (SMAC) technique have been adapted to model the flow of metal into a mold cavity. Potentially, they may be applied to the modeling of flow in ladles, tundishes, and multiple entry gating systems.

These techniques allow the flow patterns, including the location of the fluid and the velocity and pressure fields within the fluid to be calculated and plotted from the moment the fluid starts to enter the cavity, until the mold is filled, or longer if desired. From this information, the user can determine the tendency for entrapment of gas, dross, or inclusions, and for erosion of the mold.

The results of computations using these techniques have been compared with high speed motion pictures made of liquid entering transparent models of the same configurations. The agreement between the flow patterns shown by the mathematical and physical models is encouraging.

\* Originally presented and published: R. A. Stoehr and W. S. Hwang, "Modeling the Flow of Molten Metal Having a Free Surface During Entry into Molds," Modeling of Casting and Welding Processes II, Henniker, N.H., July 31 - Aug. 5, 1983.

## VII.1 COMPUTATIONAL TECHNIQUES

The Marker-and-Cell (MAC) technique was developed by Harlow et al.<sup>1</sup> of the Los Alamos Scientific Laboratory primarily for nuclear and civil engineering applications. It uses the complete Navier-Stokes equations in their primitive form including all non-linear terms. The primary dependent variables are the pressure and the velocity components. It combines the best features of the two classical computational schemes of fluid dynamics-- namely, the ideal Eulerian approach and ideal Lagrangian approach.<sup>2</sup> The cavity is divided into a large number of Eulerian cells which remain at rest. Additionally, a Lagrangian mesh of particles ("markers") represents elements of the fluid which move through the mesh of cells. The Eulerian mesh is used to characterize the fluid variables (such as velocity, pressure, etc.) while the markers are used to represent the fluid itself and define where the free surface is located. In this way, the free surface boundary may be applied. The way in which surfaces are handled particularly distinguishes the MAC method from other fluid dynamics techniques.

Up to now, the application of the MAC technique has been limited to two dimensional cases. The continuity and Navier-Stokes equations for viscous, incompressible flow in 2-D cartesian coordinates are:

$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (\text{VII.1})$$

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} = -\frac{\partial \Phi}{\partial x} + v \left\{ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right\} + g_x \quad (\text{VII.2})$$

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} = -\frac{\partial \Phi}{\partial y} + v \left\{ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right\} + g_y \quad (\text{VII.3})$$

where

$u$  and  $v$  are the velocity components in the  $x$  and  $y$  directions

$\Phi$  is the ratio of pressure to density

By the finite difference formulation of the momentum equations, the velocity components at the new time step can be calculated from the velocity components and pressure at the present time step. Using the continuity equation and forcing the divergence to equal zero at the next time step results in a Poisson type of equation which governs the pressure field.

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = -R \quad (\text{VII.4})$$

where  $R$  is an algebraic expression of the various velocity components at the present time step. An iterative procedure is used to solve Equation (VII.4) for the pressure field. This field is then used in the Navier-Stokes equations to calculate

the velocity component at the next time step. In solving Equation (VII.4), the pressure values for (imaginary) cells outside the cavity walls are needed. Since the normal velocity component always vanishes at the wall, it provides a relationship between the pressure for the (imaginary) outside cell and the pressure for its neighboring inside cell. Such boundary conditions are heterogeneous (i.e., the pressure "outside" is not equal to the pressure "inside", since they are functions of the velocities as well), but they can be calculated without ambiguity for rectangular cavities. A flow chart for the technique is shown in Figure VII-1.

Despite the successes of the MAC method, it is excessively complicated in respect to the boundary conditions in the presence of rigid obstacles and various inflow and outflow situations. The difficulties are alleviated by the Simplified Marker-and-Cell (SMAC) technique.<sup>3</sup> SMAC, employing most of the same principles as MAC, works on the principle that vorticity is independent of pressure. Pressure never needs to be calculated. An arbitrary pressure field (usually zero) is used in the finite difference momentum equations to calculate a tentative velocity field at the new time step. The tentative velocity field will have the correct vorticity but not necessarily the correct (zero) divergence. It is then assumed that a potential function  $\Psi$  exists such that the difference between the actual velocity and the tentative velocity is given by the gradient of this potential function. Combined with the continuity equation, this gives a Poisson's type of equation to govern the potential function

$$\partial^2\Psi/\partial X^2 + \partial^2\Psi/\partial Y^2 = D' \quad (\text{VII.5})$$

where  $D'$  is the divergence of the tentative velocity field.

An over-relaxation method is used to solve (VII.5) for the potential function and in turn to modify the tentative velocity field to the actual velocity field, which will have the correct vorticity and zero divergence. In this case the potential function of each outside cell is simply equal to that of its neighboring inside cell, thus simplifying its derivation and calculation. After the new velocity is calculated, it can be put back into the momentum equation to calculate the pressure if so desired.

Like most numerical techniques, MAC and SMAC suffer from numerical instability if the system properties are not properly chosen. Since the Navier-Stokes equations are nonlinear and have nonconstant coefficients the instability analysis is very complex. A heuristic method was proposed for investigating the computational instability of such finite difference equations by Hirt.<sup>4</sup> As it turns out, MAC and SMAC, like most of the numerical techniques in fluid dynamics, needs an "artificial viscosity" to stabilize the computation. From the physical point of view the flow is turbulent in most casting situations, and the Reynolds turbulent shear stress will tend

## Flow chart of FPMAC program

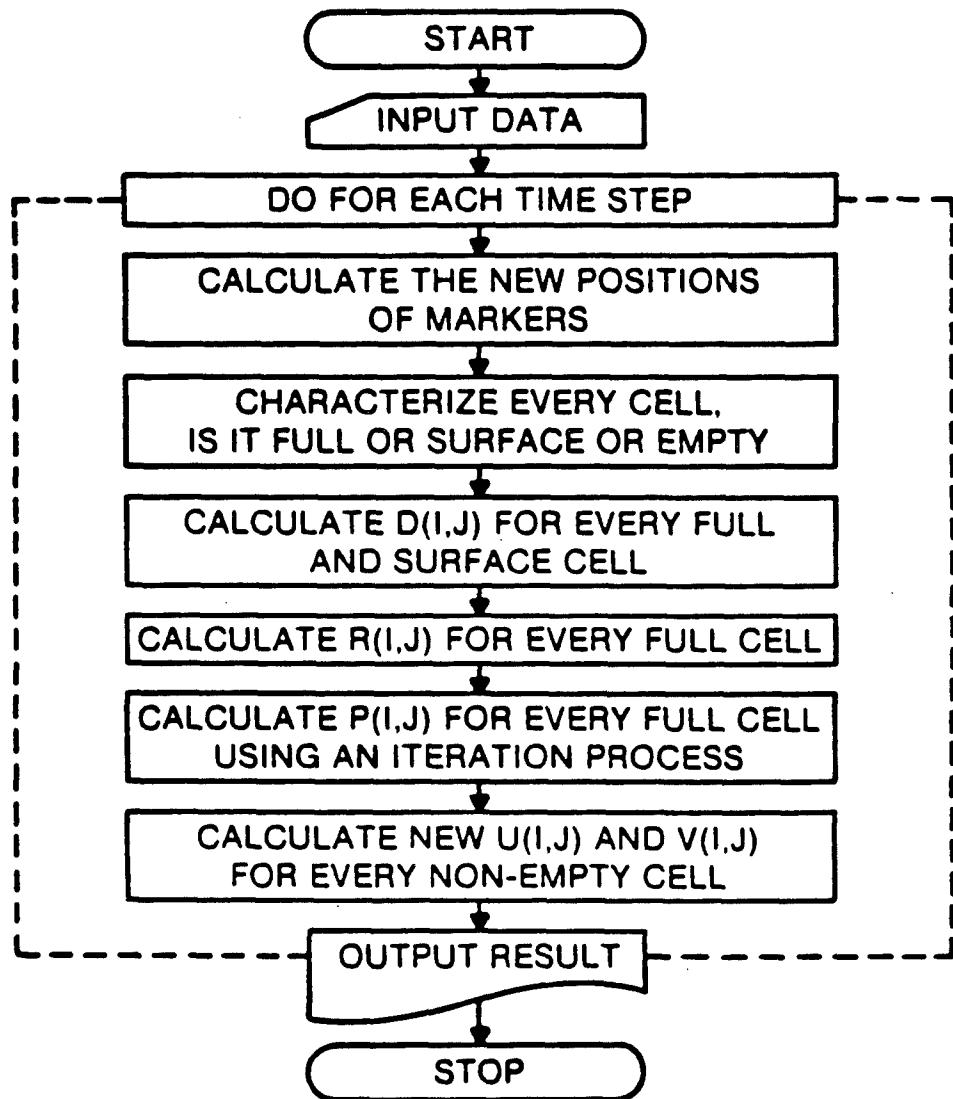


Figure VII-1: Flow chart of FPMAC program.

Table VII-1: Relation of FORTRAN and Algebraic Symbols.

FORTRAN*	Algebraic	Meaning
D(I,J)	D	Divergence
R(I,J)	R	A function of present velocities
P(I,J)	$\Phi$	Pressure/density
U(I,J)	u	x component of velocity
V(I,J)	v	y component of velocity

\* The FORTRAN subscripts I,J shown in parentheses refer to finite difference values of these variables for cell I,J.

to dominate the viscous shear stress. Thus, the artificial viscosity necessary to provide stability also represents the effective viscosity. MAC and SMAC in their present forms, however, do not utilize a turbulent shear stress model to determine the effective viscosity but choose, instead, a viscosity large enough to avoid numerical instability.

## VII.2 TEST PROBLEMS AND RESULTS

Several 2-D cases with simple square sections were simulated by the MAC technique. In the first case (Note: Figures VII-2 and VII-3), molten metal enters a horizontal plate mold through a gate in the center of the left wall at a relatively small incoming flow rate. As the metal enters the mold, the stream expands a little bit before hitting the opposite wall. When it hits the opposite wall, it divides symmetrically into two streams which flow outward along this wall, then backward along the two side walls. The two spots right next to the ingate are the last to fill, with two small vortices forming in this region.

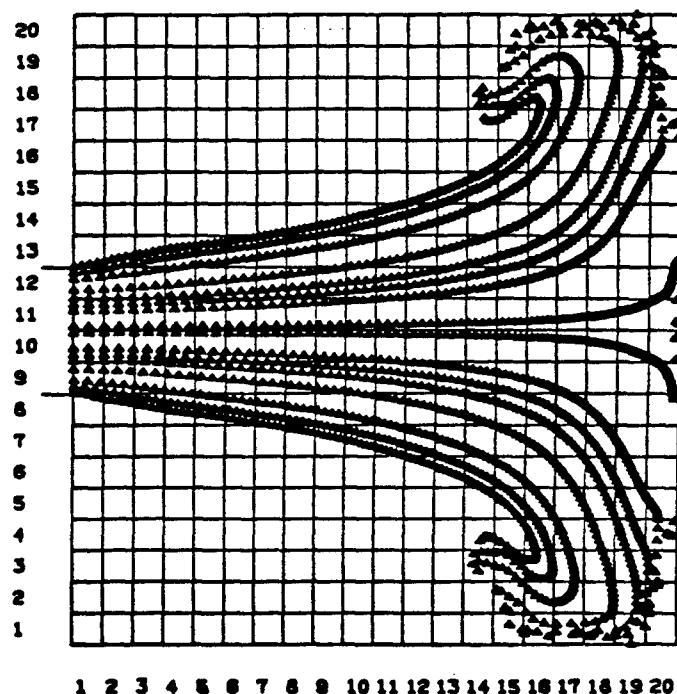
This configuration was also tried with a larger flow rate (10 times larger than the first case). The metal enters the mold like a jet. It hits the opposite wall and flows outward along it in two symmetrical streams that cling much closer to the wall than in the slower moving case. These streams race backward against the two side walls, then along the ingate wall where they encounter the incoming jet. The last areas to fill are farther away from the ingate wall than in the first case, and two large vortices develop, one on each side of the centerline.

Two types of plots are commonly made from the computed results. The first type is a "marker" plot, Figure VII-2, which shows the computed locations of the imaginary particles used to define the location of the fluid. The second is a vector plot, Figure VII-3, which shows the velocity factors for each cell which contains liquid.

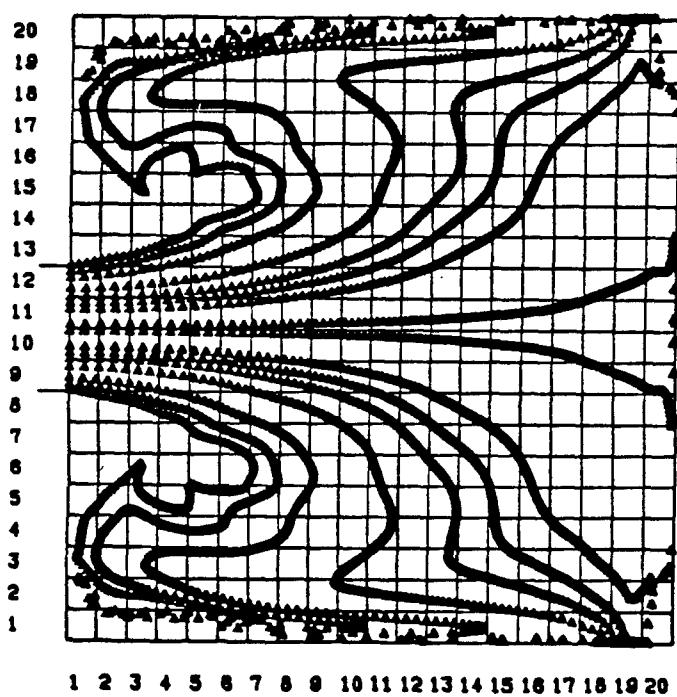
Physical verification of these flow patterns was obtained from high speed motion pictures of the entry of fluid in water - transparent plastic models. Originally, movies made at Battelle-Columbus Laboratories for the American Foundrymen's Society were used. Later, appropriate models were built and photographed at the University of Pittsburgh. These movies, showing very good correlation with the computer results, were shown with the oral presentation of this paper.

The second configuration modeled by MAC was a vertical rectangular mold in which the metal entered horizontally through a gate at the lower left corner. Verification, again was obtained by comparisons with movies of transparent models made in our laboratories as well as those of others. In addition an x-ray motion picture made of actual molten steel entering a similar shaped cavity by Ashton and Buhr the Canadian Bureau of Energy, Mines and Resources<sup>5</sup> was viewed. Agreement between the MAC predictions and the visually observable details in the movies of the physical models was quite good.

TIME: 5.45



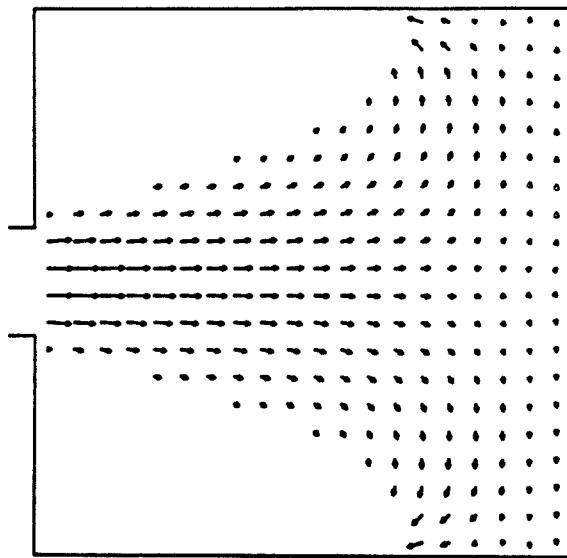
TIME: 9.95



### FLOW PATTERN

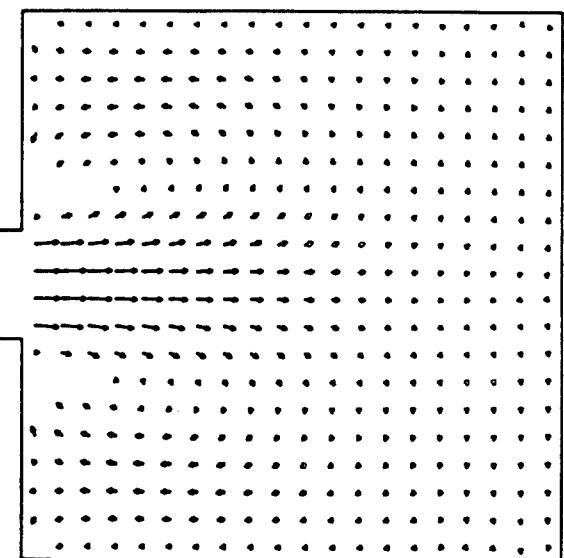
**Figure VII-2:** Marker plots of metal entering horizontal square cavity.

TIME: 5.45



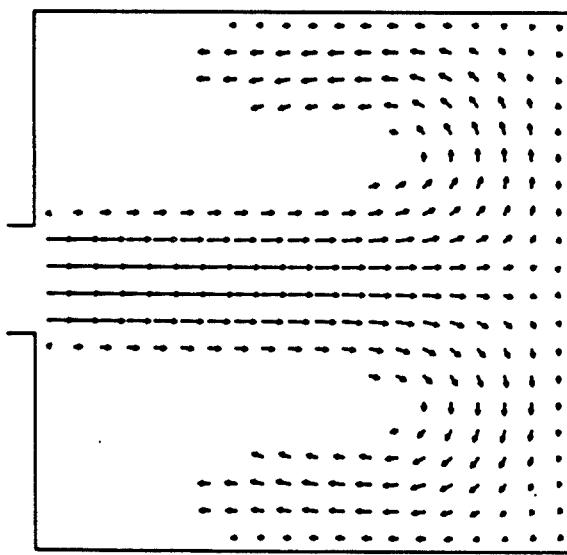
VELOCITY PROFILE

TIME: 9.95



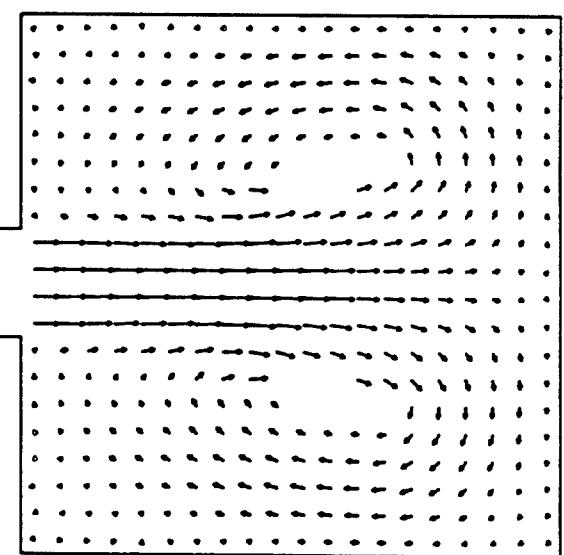
VELOCITY PROFILE

TIME: 0.599



VELOCITY PROFILE

TIME: 0.879



VELOCITY PROFILE

**Figure VII-3:** Vector plots of metal entering square cavities.

The SMAC technique was used to calculate flow into the vertical square mold with the horizontal entry of metal through a gate in the lower corner (essentially the same as the second configuration described above for MAC). The results were similar but somewhat more detailed than the MAC results. A motion picture comparing the predictions obtained with SMAC to the flow patterns recorded with the high speed camera was shown during the oral presentation. Figure VII-4 shows the vector plots for this configuration at three times during filling of the mold. Figure VII-5 shows still frames from the movie selected to show the liquid configurations and flow patterns at the same times. The agreement between them is more obvious when the movie is viewed because the location of vortices and directions of flow may be discerned more clearly than in the still frames.

Another advantage of the SMAC technique is that agreement with the incompressibility conditions is generally better than with MAC. That is, agreement between the plotted filled volume and the theoretical filled volume based on the fluid velocity and cross-section at the ingate is usually within 2 or 3 percent with SMAC, compared with 10 to 12 percent with MAC.

The SMAC technique was also used to calculate flow into a mold for a horizontal stepped-plate casting. The vector plots at two time steps are shown in Figure VII-6. These plots show how standing waves develop at the steps, and how fluid touches the top of the cavity above the first step before any fluid reaches the far end of the mold. This configuration, with its re-entrant angles at the steps would be relatively difficult to model with MAC, but it is well suited to the SMAC technique. These flow patterns, including the location and duration of standing waves and vortices, were essentially verified by high speed movies of the transparent models.

This technique was also used to model flow in a complete system consisting of a pouring basin, down sprue, sprue extension, horizontal runner, and vertical square mold cavity as shown in Figure VII-7. Initially, all of the metal was retained in the pouring basin by a stopper at the top of the down sprue. At the start of the pour, the stopper was removed, allowing liquid to enter the sprue and flow on into the casting cavity. The pouring basin was maintained full of metal throughout the pour. At the time step shown in Figure VII-7a, the sprue, which is not tapered, is still only partially filled. The system is not "pressurized", and the metal is free falling through the sprue. At a later time, shown in Figure VII-7b, the sprue is full and waves are seen on the surface of the metal in the mold cavity. This demonstrates the ability of these techniques to model fairly complex systems.

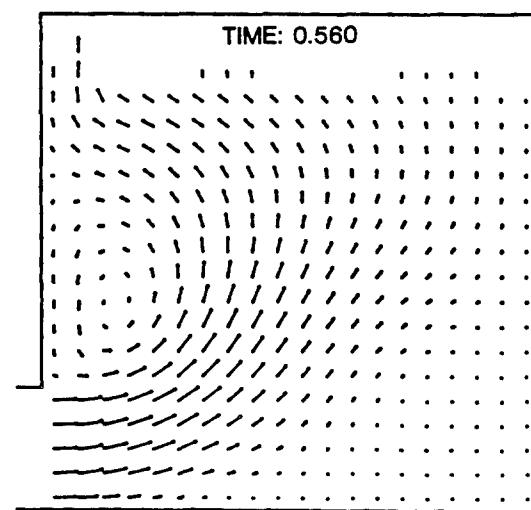
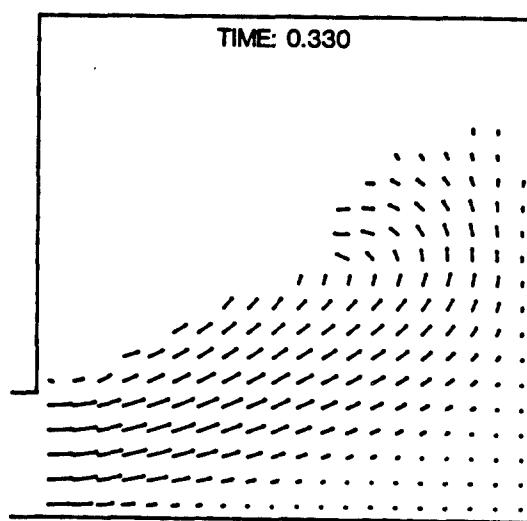
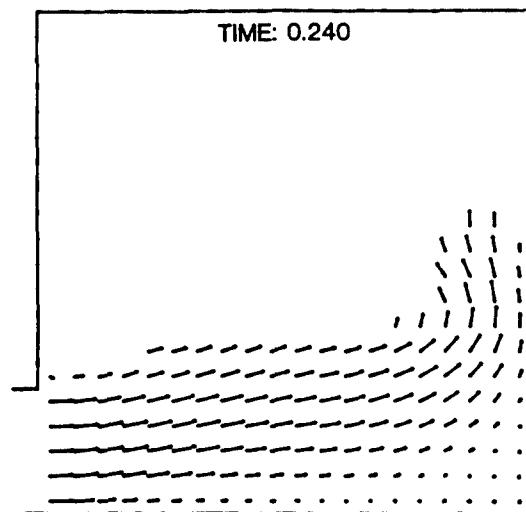
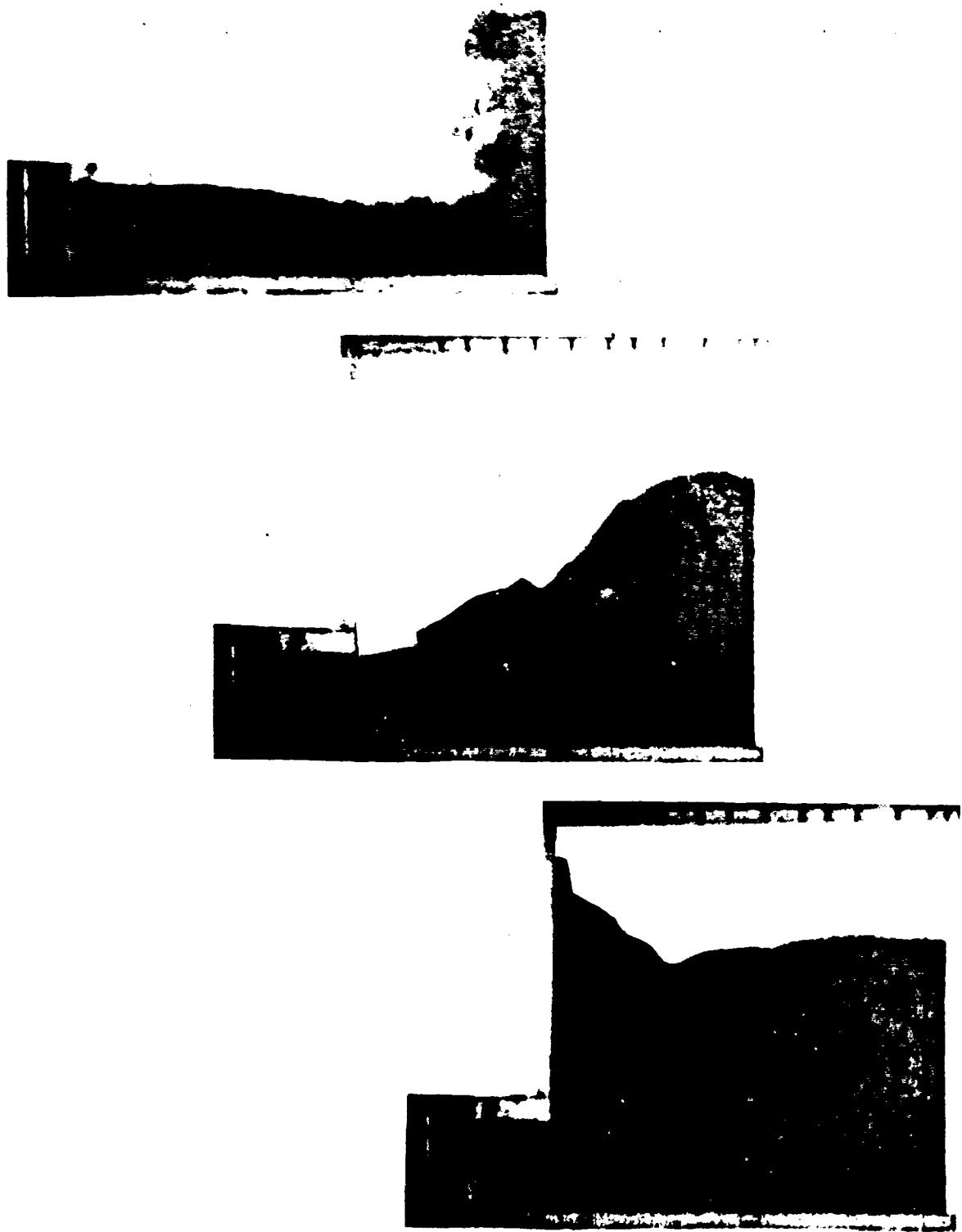
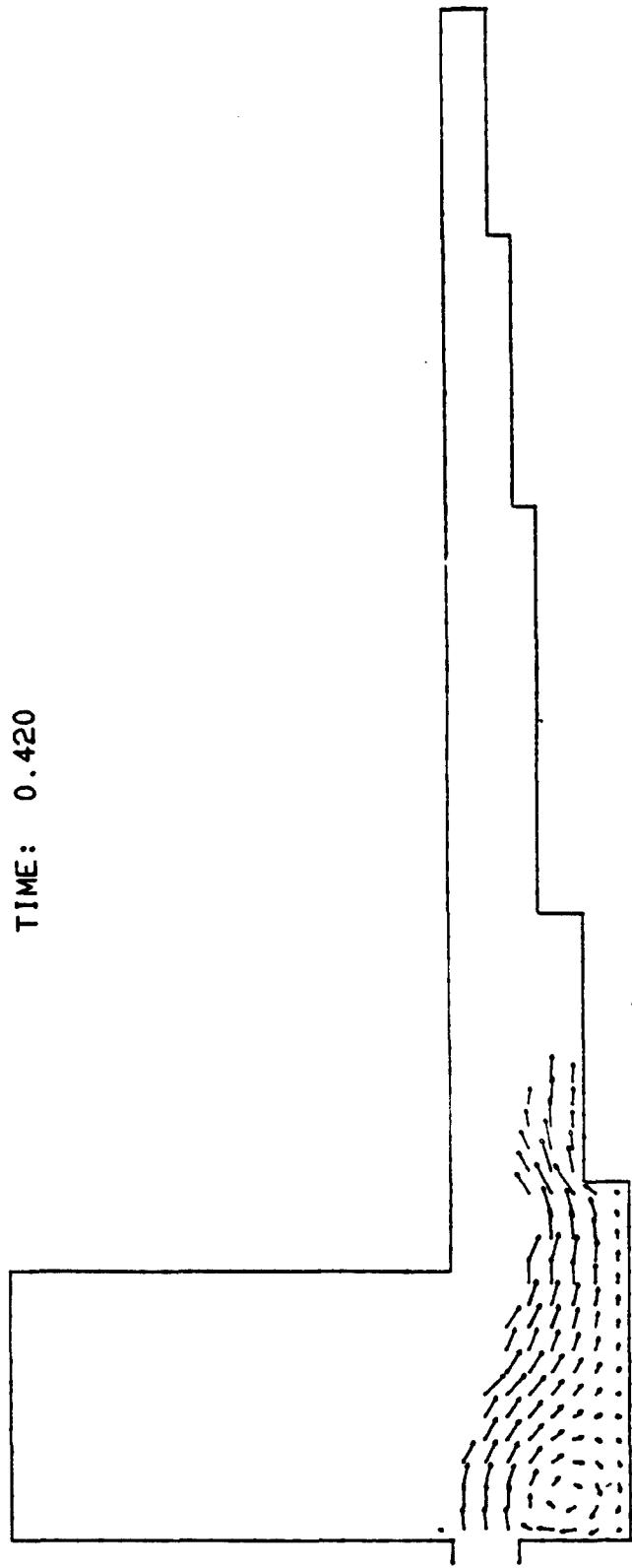


Figure VII-4: Vector plots of flow into vertical mold, with SMAC.



**Figure VII-5:** High-speed movie frames of flow into a vertical mold.

TIME: 0.420



TIME: 1.260

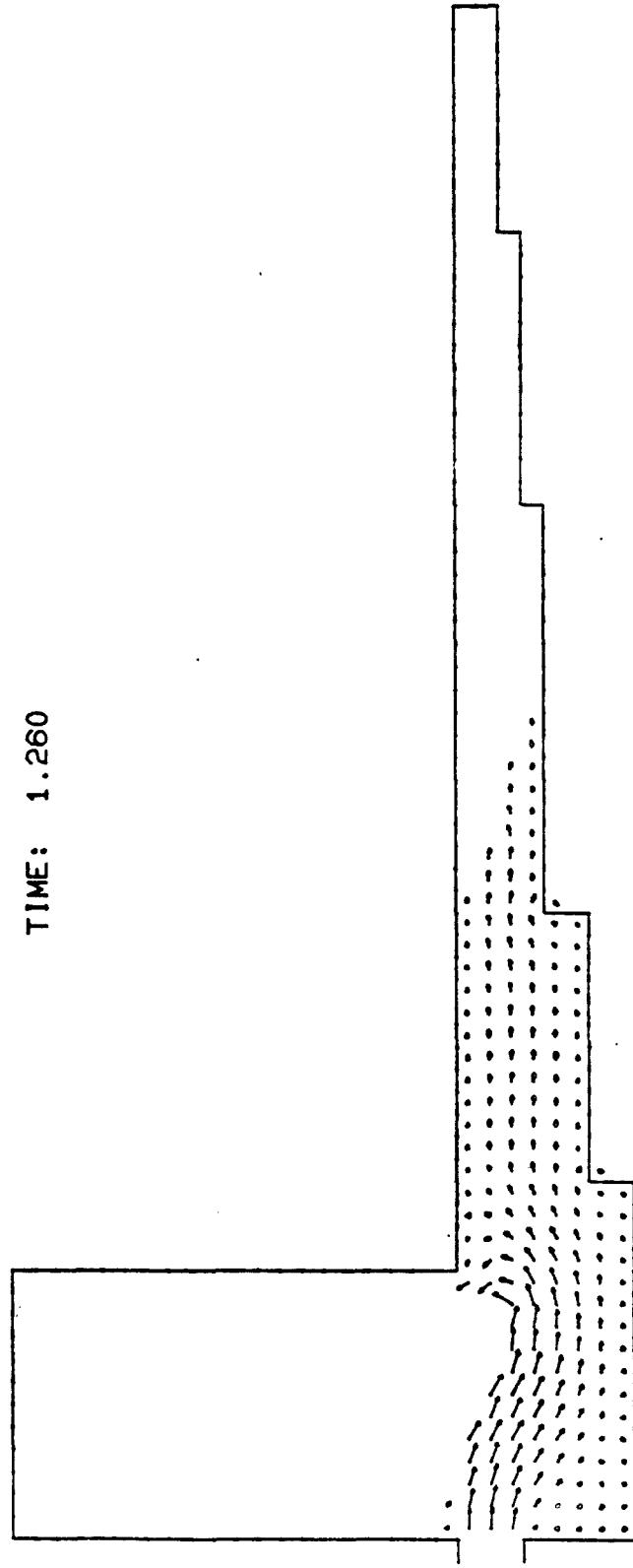


Figure VII-6: Vector plots, entry to stepped mold cavity, with SMAC.

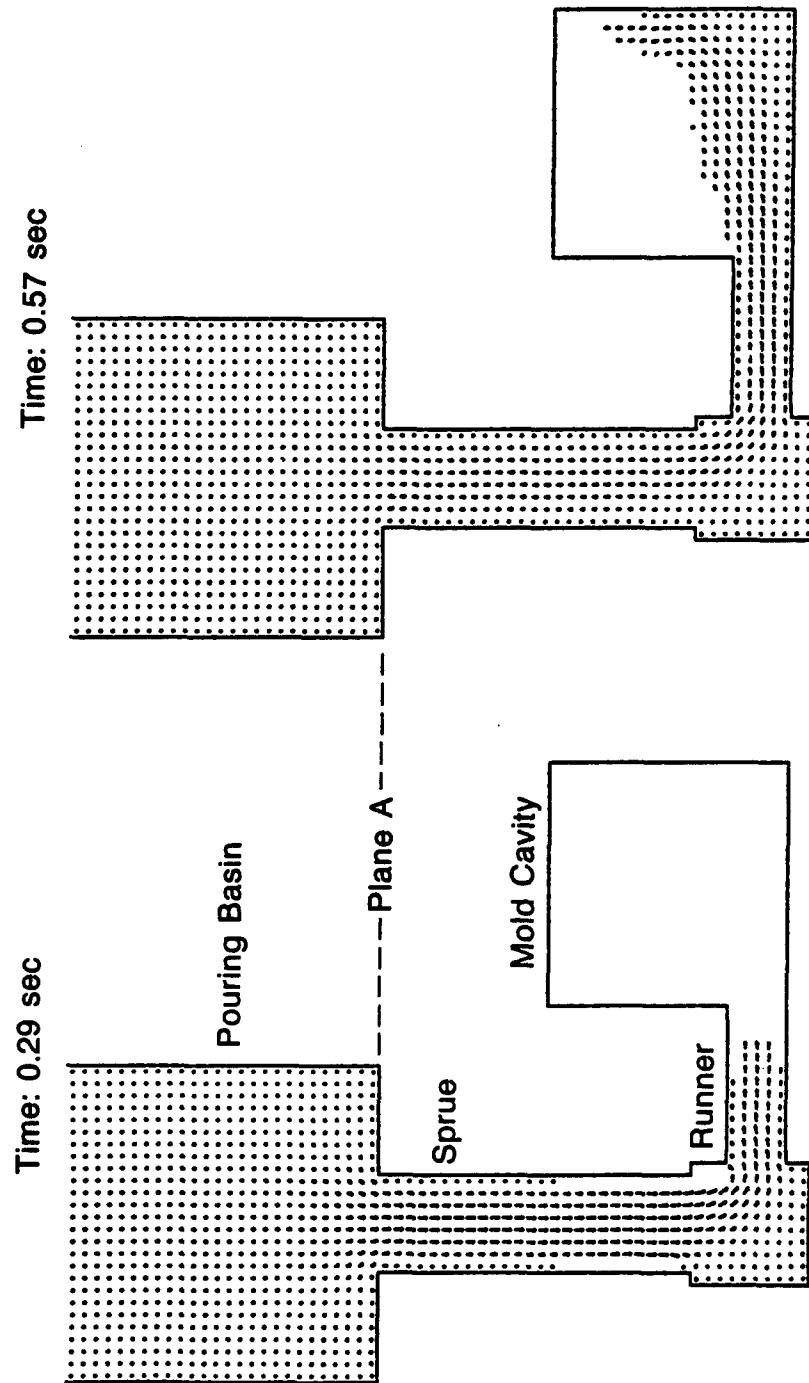


Figure VII-7: Vector plot for complete casting system, with SMAC.

### **VII.3 DISCUSSION AND CONCLUSIONS**

It is believed that MAC and SMAC provide very useful tools to look into a wide variety of metallurgical operations which previously could only be studied by experiment. The present application is limited to mold-filling, but by a little extension another part of the casting process, the design of the gating system, could also utilize this kind of analysis. Through these techniques the designer could use numerical simulation, rather than physical experiment, to design a proper system to minimize turbulence, avoid air entrapment, provide for separation of dross and inclusions from the metal, and produce the desired distribution of flows through the multiple ingates of larger systems. It also has the potential for being coupled with heat flow calculations to determine the actual temperature distribution in the mold during and immediately after filling. Further efforts to extend the application of these techniques to such metallurgical problems appears to be well justified.

#### **ACKNOWLEDGEMENTS**

This work has been performed as part of Contract No. DAAK-30-70-C-108 from the U.S. Army Tank-Automotive Command, Research and Development Center, Warren, Michigan, Mr. Michael Holly, project monitor. Professor Harold D. Brody, University of Pittsburgh, has been the project director. Miss Shiao-Fong Yin and Mr. Robert Pegnato contributed substantially to the building and photographing of the transparent plastic models while students at the University of Pittsburgh.

## REFERENCES

1. J. E. Welch, F. H. Harlow, J. P. Shannon and B. J. Bally, "The MAC Method, A Computing Technique for Solving Viscous, Incompressible, Transient Fluid-flow Problems Involving Free Surfaces," Techn. Report LA-3425, Los Alamos Scientific Laboratory, 1966.
2. B. Alder, S. Fernbach and M. Rotenberg, Methods in Computational Physics, Academic Press, New York, N. Y., Volume 3, 1964.
3. A. A. Amsden and F. H. Harlow, "The SMAC Method, A Numerical Technique for Calculating Incompressible Fluid Flows," Techn. Report LA-4370, Los Alamos Scientific Laboratory, 1970.
4. C. W. Hirt, "Heuristic Stability Theory for Finite-Difference Equations", Journal of Computational Physics, Vol. 2, pp. 339-355, (1968).
5. M. C. Ashton and R. K. Buhr, "Direct Observation of the Flow of Molten Steel in Sand Molds," Techn. Report TM-M-73-5, CANMET, Energy, Mines and Resources Canada, 1973.

DISTRIBUTION LIST

	<u>COPIES</u>
Defense Technical Information Center ATTN: TIPDR Cameron Station Alexandria, VA 22314	2
Commander U. S. Army Materiel Command ATTN: AMCMT-M 5001 Eisenhower Ave Alexandria, VA 22333-0001	1
Director U. S. Army Materials Technology Lab ATTN: SLCMT-D Watertown, MA 02172	2
Director U. S. Army Industrial Base Engineering Activity ATTN: AMXIB-MT Rock Island Arsenal Rock Island, IL 61299	2
Commander U. S. Army Materiel Industrial Resources Offices ATTN: Code 044 Philadelphia, PA 19112	1
Commander Air Force Materials Laboratory ATTN: LTM Wright Patterson AFB, OH 45433	1
Commander U. S. Army Tank-Automotive Command ATTN: AMSTA-TSL Warren, MI 48397-5000	12

Commander 1  
U. S. Army Research Office  
ATTN: Chief, IPO  
P. O. Box 12211  
Research Triangle Park, N. C. 27709

Manager 2  
Defense Logistics Studies  
Information Exchange  
ATTN: AMXMC-D  
Ft Lee, VA 23801-6044

Commander 1  
U. S. Army Tank-Automotive Command  
ATTN: AMSTA-CF, Mr. G. Orlicki  
Warren, MI 48397-5000

Commander 2  
U. S. Army Aviation R & D Command  
ATTN: DRDAV-QF, Mr. John Conroy  
4300 Goodfellow Blvd.  
St. Louis, MO 63120

Commander 1  
Harry Diamond Laboratories  
ATTN: Library  
Washington, D. C. 20438

National Science Foundation 2  
ATTN: Mr. A. Schwarzkopf  
1STI, Room 1121  
Washington, D. C. 20550

Steel Founders Society of America 1  
Cast Metals Federation Bldg.  
20611 Center Ridge Road  
Rocky River, OH 44116

University of Wisconsin  
Metallurgical and Mineral Engineering  
ATTN: Dr. Heine  
1509 University Ave  
Madison, WI 53706

1

University of Alabama  
Dept. of Metallurgical Engineering  
ATTN: Dr. John T. Berry  
University, Alabama 35486

1

The University of Michigan  
Materials and Metallurgical Engineering  
ATTN: Dr. R. D. Pehlke  
Ann Arbor, MI 48109

1

University of Maryland  
College of Engineering  
ATTN: Frank F. Moderacki  
Frostburg, MD 21532

1

ABEX Corporation  
Research Center  
ATTN: Mr. M. C. Meslink  
Mahwah, N. J. 07430

1